Fix an elliptic curve $E$ over $\mathbb{Q}$. An “extremal prime” for $E$ is a prime $p$ of good reduction such that the number of rational points on $E$ modulo $p$ is maximal or minimal in relation to the Hasse bound. In this talk, I will discuss what is known and conjectured about the number of extremal primes $p \leq X$, and give the first non-trivial upper bound for the number of such primes when $E$ is a curve without complex multiplication. The result is conditional on the hypothesis that all the symmetric power $L$-functions associated to $E$ are automorphic and satisfy the Generalized Riemann Hypothesis. In order to obtain this bound, we use explicit equidistribution for the Sato-Tate measure as in recent work of Rouse and Thorner, and refine certain intermediate estimates taking advantage of the fact that extremal primes have a very small Sato-Tate measure.