
MAKSYM RADZIWIŁ, California Institute of Technology

Recent progress in multiplicative number theory

Multiplicative number theory aims to understand the ways in which integers factorize, and the distribution of integers with special multiplicative properties (such as primes). It is a central area of analytic number theory with various connections to L-functions, harmonic analysis, combinatorics, probability etc. At the core of the subject lie difficult questions such as the Riemann Hypothesis, and they set a benchmark for its accomplishments.

An outstanding challenge in this field is to understand the multiplicative properties of integers linked by additive conditions, for instance n and $n + 1$. A central conjecture making this precise is the Chowla-Elliott conjecture on correlations of multiplicative functions evaluated at consecutive integers. Until recently this conjecture appeared completely out of reach and was thought to be at least as difficult as showing the existence of infinitely many twin primes. These are also the kind of questions that lie beyond the capability of the Riemann Hypothesis.

However recently the landscape of multiplicative number theory has been changing and we are no longer so certain about the limitations of our (new) tools. I will discuss the recent progress on these questions.