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A New Technique for Constructing Exponential Integrators

A new method is proposed for deriving exponential integrators for stiff ordinary differential equations. Exponential integrators are explicit discretizations that solve the linear part exactly. An important property of exponential integrators is that they reduce to classical discretizations in the limit of vanishing linearity. We review the history and current status of exponential integrators, which have been rediscovered many times since Certaine first used an exponential method in 1960.

While first- and second-order exponential integrators are well understood, higher-order methods are difficult to construct. We demonstrate a technique that can circumvent this problem, converting classical integration methods into corresponding exponential discretizations.

Stiff-order conditions for exponential integrators were developed in a Banach space framework by Hochbruck and Ostermann [2005]. They showed that in the worst case, the well-known ETDRK4 integrator of Cox and Matthews can exhibit an order reduction from four to two. Although they speculated that a four-stage exponential integrator with stiff-order four does not exist, we have nevertheless succeeded in constructing a four-stage exponential version of the classical RK4 integrator with stiff-order four. Our ERK4 integrator performs well in numerical tests and generalizes to the case where the dependent variable is a vector and the linear operator is a diagonal matrix. We plan to extend our construction to the case where the linear operator is a general square matrix. We also wish to derive embedded exponential Runge-Kutta pairs that efficiently generate both a high- and low-order estimate, allowing dynamic adjustment of the time step to achieve a specified accuracy.