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Algebraic dual polynomials for mimetic spectral element methods

In this work the High Order Mimetic Discretization Framework will be presented together with a construction of algebraic dual polynomials.

The degrees of freedom are associated to geometric objects, and a relation to differential forms will be remarked. We show how to construct discrete polynomial function spaces of arbitrary degree associated to these geometric degrees of freedom, constituting a discrete de Rham complex:

$$\mathbb{R} \longrightarrow V_h^0 \subseteq H(\nabla,\Omega) \xrightarrow{\nabla} V_h^1 \subseteq H(\nabla\times,\Omega) \xrightarrow{\nabla\times} V_h^2 \subseteq H(\nabla\cdot,\Omega) \xrightarrow{\nabla\cdot} V_h^3 \subseteq L^2(\Omega) \longrightarrow 0\,.$$

In this way it is possible to exactly discretize topological equations even on highly deformed meshes.

An important aspect in the numerical solution of partial differential equations is the efficient construction of the system matrix and its subsequent solution. Given a polynomial basis Ψ_i which spans the polynomial vector space $\mathcal{P}$, this work addresses the construction and use of the algebraic dual space $\mathcal{P}'$ and its canonical basis. It will be shown that a primal-dual formulation using these algebraic dual basis results in a very sparse system matrix where two of the sub-matrices consist of only incidence matrices. The method will be applied to a Dirichlet and Neumann problem and it is shown that the finite dimensional approximations satisfy $\phi^h = \nabla\cdot\boldsymbol{q}^h$ on any grid. The dual method is also applied to a constrained minimization problem, which leads to a mixed finite element formulation. The discretization of the constraint and the Lagrange multiplier will be independent of the grid size, grid shape and the polynomial degree of the basis functions.