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Asymptotics of Fundamental Solutions for Helmholtz operators on Spaces of Constant Curvature

We compute closed-form expressions for oscillatory and damped spherically symmetric fundamental solutions of the Helmholtz equation in d-dimensional hyperbolic and hyperspherical geometry. We are using the R-radius hypersphere and R-radius hyperboloid model of hyperbolic geometry. These models represent Riemannian manifolds with positive constant and negative constant sectional curvature respectively. Flat-space limits with their corresponding asymptotic representations, are used to restrict proportionality constants for these fundamental solutions. In order to accomplish this, we summarize and derive new large degree asymptotics for associated Legendre and Ferrers functions of the first and second kind. Furthermore, we prove that our fundamental solutions on the hyperboloid are unique due to their decay at infinity. To derive Gegenbauer polynomial expansions of our fundamental solutions for Helmholtz operators on hyperspheres and hyperboloids, we derive a collection of infinite series addition theorems for Ferrers and associated Legendre functions which are generalizations and extensions of the addition theorem for Gegenbauer polynomials. Using these addition theorems, in geodesic polar coordinates for dimensions greater than or equal to three, we compute Gegenbauer polynomial expansions for these fundamental solutions, and azimuthal Fourier expansions in two-dimensions.