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Large locally finite-dimensional operator algebras

A C*-algebra A is said to be locally finite-dimensional (LF) if finite subsets of A can be approximated from finite-dimensional subalgebras. It is called approximatelly finite-dimensional (AF) if it contains a directed family of finite-dimensional subalgebras whose union is dense. These two notions coincide for separable C*-algebras (Bratteli) but are different in general (Farah, Katsura). A C*-algebra is called scattered if each of its subalgebras is LF. This generalizes $C(K)$ for K scattered, i.e., where every subset has an isolated point. We survey our recent joint results with T. Bice and with S. Ghasemi concerning some locally finite-dimensional nonseparable operator algebras in $B(\ell_2)$. The results include:

There are C*-subalgebras of $B(\ell_2)$ with the following properties:

1) A nonseparable AF algebra with no nonseparable abelian C*-subalgebra,

2) An extension of $K(\ell_2(2^\omega))$ by $K(\ell_2)$ which is not stable,

3) An inductive limit of an increasing system of separable stable AF algebras which is not stable,

There are C*-algebras with the following properties, and the existence of such operator algebras in $B(\ell_2)$ is undecidable:

4) A scattered C*-algebra which is not AF,

5) An extension of an AF algebra by AF algebra which is not AF,

6) An LF algebra which is not AF,

7) An algebra which does not have $\ll$-increasing approximate unit ($a \ll b$ iff $a = ab$),

All the above algebras are scattered. The methods are inspired by commutative combinatorial set theory (trees, almost disjoint families, gaps, Q-sets) and often mix with logic.