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Prime number races with many contestants

We investigate the logarithmic densities in prime number races with r competitors modulo q, when $r, q \to \infty$, assuming the standard conjectures GRH and LI. Among our results, we uncover an interesting transition in the asymptotic behavior of these densities when $r = (\log q)^{1+o(1)}$. First, in a joint work with A. Harper, we prove that these densities are all asymptotic to $1/r!$ when $r \leq (\log q)^{1-\epsilon}$, thus showing that all biases dissolve in this range. On the other hand, in a recent joint work with K. Ford and A. Harper, we show that when $r/\log q \to \infty$, there exist r-way prime number races where the densities are much smaller than $1/r!$, and others where the densities are much larger than $1/r!$, answering a question of A. Feuerverger and G. Martin. The proofs use various probabilistic tools, including a version of Stein's method of exchangeable pairs, and a quantitative multidimensional Gaussian approximation theorem, obtained through Lindeberg's method.