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*Nested efficient congruencing and relatives of Vinogradov's mean value theorem*

The main conjecture in Vinogradov's mean value theorem states that, for each  $\epsilon > 0$ , one has

$$\int_{[0,1]^k} \left| \sum_{1 \leq n \leq X} e(\alpha_1 x + \dots + \alpha_k x^k) \right|^{2s} d\alpha \ll X^{s+\epsilon} + X^{2s-k(k+1)/2}.$$

This is now a theorem of Bourgain, Demeter and Guth (in 2016, via  $l^2$ -decoupling) and the speaker (in 2014 for  $k=3$ , and in 2017 in general, via (nested) efficient congruencing). We report on some generalisations of this conclusion, some of which go beyond the orbit of decoupling and efficient congruencing.