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Derivatives and special values of higher-order Tornheim zeta functions

We study analytic properties of the Tornheim zeta function $T(r, s, t)$, and in particular the case $\omega_3(s) := T(s, s, s)$. While the values at positive integers have long been known, we evaluate $\omega_3(0)$ and show that $\omega_3(m) = 0$ for all negative integers m. As our main result, we find the derivative of this function at $s = 0$, which turns out to be surprisingly simple. I will also show that all these results have analogues for Tornheim zeta functions of arbitrary orders. These results were first conjectured by J. Borwein and D. Bailey using high-precision calculations based on an identity due to R. Crandall that involves a free parameter and provides an analytic continuation. This identity was also the main tool in the eventual proofs of our results. (Joint work with Hayley Tomkins).