GEORGE ELLIOTT, University of Toronto
A simple report on simple C*-algebras

The classification of simple separable amenable C*-algebras which are Jiang-Su stable and satisfy the Universal Coefficient Theorem (UCT) is almost complete—and is complete in the unital case. In the infinite case (unital or not) it was complete with the work of Kirchberg and Phillips (and earlier authors) twenty years ago.

In the non-unital case a semi-abstract setting has been dealt with that may contain all examples. In the abstract non-unital setting it is still not known if Jiang-Su stability implies finite nuclear dimension (an essential step in the unital case), or for that matter if finite nuclear dimension implies that the semi-abstract axioms (analogous to TAF, etc., in the unital case) are satisfied. The latter implication is known in the case of trivial K-groups, and so these algebras (i.e., simple separable amenable UCT C*-algebras with finite nuclear dimension and K-groups equal to zero) are classified by their tracial cone, together the norm function (the norm of a densely defined lower semicontinuous trace may be finite or infinite).

Note that the tensor product of any C*-algebra with the Jiang-Su C*-algebra is Jiang-Su stable, and is simple, separable, and amenable if the given one is, and so there are many examples of Jiang-Su stable simple separable amenable C*-algebras. Sometimes, Jiang-Su stability holds from the beginning (without tensoring!). Sometimes, this is less evident, but the property of finite nuclear dimension (which is in any case the operative property for classification, and in particular is known to imply Jiang-Su stability) can be established directly.