Generalized Stieltjes constants occur as coefficients of \((s - 1)^k\) in the Laurent series expansion of certain Dirichlet series around \(s = 1\). The connection between these generalized Stieltjes constants and derivatives of \(L(s, f)\) for periodic arithmetical functions \(f\), at \(s = 1\) is known. We utilize this link to throw light on the arithmetic nature of \(L'(1, f)\) and certain Stieltjes constants. In particular, if \(p\) is an odd prime greater than 7, then we deduce the transcendence of at least \((p - 7)/2\) of the generalized Stieltjes constants, \(\{\gamma_1(a, p) : 1 \leq a < p\}\), conditional on a conjecture of S. Gun, M. Ram Murty and P. Rath.