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The distribution of the number of subgroups of the multiplicative group

Let $I(n)$ denote the number of isomorphism classes of subgroups of $(\mathbb{Z}/n\mathbb{Z})^\times$, and let $G(n)$ denote the number of subgroups of $(\mathbb{Z}/n\mathbb{Z})^\times$ counted as sets (not up to isomorphism). We prove that both $\log G(n)$ and $\log I(n)$ satisfy Erdős–Kac laws, in that suitable normalizations of them are normally distributed in the limit. Of note is that $\log G(n)$ is not an additive function but is closely related to the sum of squares of additive functions. We also establish the orders of magnitude of the maximal orders of $\log G(n)$ and $\log I(n)$.