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LSLQ: An Iterative Method for Linear Least-Squares Problems with a Forward Error Minimization Property

We propose an iterative method named LSLQ for solving consistent linear systems or linear least-squares problems $Ax \approx b$ of any shape based on the Golub-Kahan process, where the dominant cost consists in products with A and A^T. In the rank deficient case, LSLQ identifies the minimum least-squares solution. LSLQ is formally equivalent to SYMMLQ (Paige and Saunders, 1975) applied to the normal equations so that the estimate norm $\|x_k\|_2$ increases monotonically and the forward error $\|x_k - x^*\|_2$ decreases monotonically. We provide lower and upper bound estimates on the forward error along the LSLQ iterations. The upper bound translates to an upper bound on the forward error in the Euclidean norm for LSQR, which was previously unavailable. We report numerical experiments on standard test problems and on a full-wave inversion problem arising from geophysics in which an approximate least-squares solution corresponds to an approximate gradient of a relevant penalty function that is to be minimized.

(joint work with Ron Estrin (ICME, Stanford) and Michael A. Saunders (ICME, Stanford))