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Approximating Min-cost Chain-constrained Spanning Trees: A Reduction From Weighted To Unweighted Problems

We consider the problem of finding a min-cost spanning tree satisfying degree bounds for a nested family (i.e., a chain) of node-sets. We give the *first* approximation algorithm for this problem that approximates *both* the cost and degree bounds by a constant factor. This is also the first result that obtains an constant-factor approximation for both the cost and degree bounds for any spanning-tree problem with general degree bounds on node sets, where an edge participates in a non-constant number of degree constraints. Our algorithm is based on a novel use of *Lagrangian duality* to simplify the *cost structure* of the underlying problem to obtain a decomposition into uniform-cost subproblems, and then using a known algorithm for the unweighted problem. We show that this Lagrangian-relaxation based idea is in fact applicable more generally and, for any cost-minimization problem with packing side-constraints, yields a reduction from the weighted to the unweighted problem. We believe that this reduction is of independent interest. As another application of our technique, we consider the k-*budgeted matroid basis* problem, where we leverage a recent rounding algorithm for the problem to obtain an improved $n^{O(k^{1.5}/\epsilon)}$-time algorithm that returns a solution that satisfies (any) one of the budget constraints exactly and incurs a $(1+\epsilon)$-violation of the other budget constraints.