Combinatorial Algebraic Geometry
Géométrie algébrique combinatoire
(Org: **Kiumars Kaveh** (University of Pittsburgh) and/et **Frank Sottile** (Texas A&M University))

HIRAKU ABE, McMaster University / Osaka City University Advanced Mathematical Institute
A Weyl character formula for Hessenberg varieties

Hessenberg varieties are defined to be subvarieties of the full flag variety, and hence the line bundles on the full flag variety restrict on Hessenberg varieties. In this talk, I will discuss a Weyl type character formula for the torus action on the space of global sections of these line bundles on a regular semisimple Hessenberg variety. This is a work in progress with Naoki Fujita and Jeremy Lane.

H. PRAISE ADEYEMO, Fields Institute, Toronto Canada
Equivariant Cohomology Theories and The Pattern Map

Billey and Braden defined a geometric pattern map on flag manifolds which extends the generalized pattern map of Billey an Postnikov on Weyl groups. The interaction of this torus equivariant map with the Bruhat order and its action on line bundles lead to formulas for its pull back on the equivariant cohomology ring and on equivariant K-theory. These formulas are in terms of the Borel presentation, the basis of Schubert, and localization at torus fixed points. This is joint work with Frank Sottile

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BARBARA BOLOGNESE, Fields Institute
On the connectivity of dual graphs of projective curves

In 1962, Hartshorne proved that the dual graphs of an arithmetically Cohen- Macaulay scheme is connected. After establishing a correspondence between the languages of algebraic geometry, commutative algebra and combinatorics, we are going to refine Hartshorne's result and measure the connectedness of the dual graphs of certain projective schemes in terms of an algebro-geometric invariant of the projective schemes themselves, namely their Castelnuovo-Mumford regularity. This is joint work with B. Benedetti and M. Varbaro.

LAURA ESCOBAR, The Fields Institute and University of Illinois at Urbana Champaign
The multidegree of the multi-image variety

The multi-image variety is a subvariety of $\mathrm{Gr}(2,4)^n$ that models taking pictures with n rational cameras. We compute its cohomology class in the cohomology ring of $\mathrm{Gr}(2,4)^n$ and its multidegree in the Plücker embedding $(\mathbb{P}^5)^n$. Joint work with Allen Knutson.

ANGELA GIBNEY, University of Georgia
Combinatorial aspects of conformal blocks on the moduli space of curves

In this talk I will give a low-tech definition of vector bundles of conformal blocks on the moduli space of curves, and explain why the bundles are interesting to people who study moduli of curves and moduli of vector bundles on curves. I will illustrate the combinatorial interplay between bundles of conformal blocks and the moduli spaces of curves and vector bundles on curves through a sample of results.

NATHAN ILTEN, Simon Fraser University
Dual curves, Newton polygons, and tropicalization

Classical formulas of Plücker and Noether dictate how the degree of the projective dual C^* of a plane curve C depends on the degree of C and its singularities. In this talk, I will consider a more refined invariant: the Newton polygon. Given a polygon P and any sufficiently generic plane curve C with Newton polygon P, I will show how the Newton polygon of C^* can determined solely from the combinatorics of P. Our main tools are tropical geometry, and the notion of the projective dual of a tropical plane curve. This is joint work in progress with Yoav Len, Bernd Schober, and Kristin Shaw.

LARS KASTNER, The Fields Institute
Ext and Tor on two-dimensional cyclic quotient singularities

The geometry of two-dimensional cyclic quotient singularities is deeply connected with the associated continued fractions, as discovered by Riemenschneider. This connection has been exploited for studying e.g. the monodromy and deformations of cyclic quotient singularities. In this talk we will show how this connection appears when computing Ext of two torus invariant Weil divisors on a cyclic quotient singularity. If one uses the Tor functor instead of Ext, one observes almost the same structure as for Ext. This leads to a new connection between Ext and Tor, thereby also connecting Tor with the associated continued fractions.

As an application one can compute generators of the global sections of the sheaf of a torus-invariant Weil divisor from the continued fraction.

ASKOLD KHOVANSKII, University of Toronto
RESULTANT OF LAURANT POLYNOMIALS WHOSE NEWTON POLYHEDRA ARE DEVELOPED

My talk is based on a joint work with Leonid Monin.

A system of n equations in $(\mathbb{C}^*)^n$ whose Newton polyhedra are developed (that is, they are in general position relative to each other) in many ways, resembles an equation in one unknown. As in the one-dimensional case, one can explicitly compute: 1) the sum of values of any Laurent polynomial over the roots of the system; 2) the product of all of the roots of the system (regarded as elements in the group $(\mathbb{C}^*)^n$). We study the resultant R (defined up to a sign) of an $(n+1)$-tuple of Laurent polynomials $P_1, ..., P_{n+1}$, such that for any n-tuple of them, the corresponding Newton polyhedra are developed. One can show that in this case $R = \pm Q_i M_i$ for any $1 \leq i \leq n$, where Q_i is the product of P_i over the common zeros of the P_j, for $j \neq i$, and M_i is a certain monomial in the coefficients of all the Laurent polynomials P_j with $j \neq i$. Thus the identity

$$Q_i M_i = Q_j M_j (-1)^{f(i,j)}$$

for some $f(i,j) \in \mathbb{Z}/2\mathbb{Z}$ holds. We find *explicit formulas for the monomials* M_i, M_j *and for the sign* $(-1)^f(i,j)$. The identity above make sense by itself (without mentioning the resultant). One can give an explicit algorithm for computing the products Q_k (for any $1 \leq k \leq n+1$). Hence we get *an explicit algorithm for computing the resultant* R.

YOAV LEN, University of Waterloo
A tropical Clifford's theorem

I will discuss several tropical versions of classical results concerning special divisors on curves. I will show that tropical curves, and more generally metrized complexes, satisfy Clifford's theorem in its full generality. That is, having a special divisor whose rank equals half the degree is not only necessary, but in fact sufficient for hyperellipticity. I will consider other classical characterizations for hyperellipticity, and show that they don't carry well into the tropical world. When a tropical curve is already known to be hyperelliptic, I will provide a full description of its special divisors.

DIANE MACLAGAN, University of Warwick
Tree compactifications of the moduli space of genus zero curves

The moduli space $M_{0,n}$ of smooth genus zero curves with n marked points has a standard compactification by the Deligne-Mumford module space of stable genus zero curves with n marked points. The compactification can be constructed as the closure of $M_{0,n}$ inside a toric variety. The fan of the toric variety is moduli space of phylogenetic trees. I will discuss joint work with Dustin Cartwright to construct other compactifications of $M_{0,n}$ by varying the toric variety using variants of phylogenetic trees. These compactifications include many of the standard alternative compactifications of $M_{0,n}$.

LEONID MONIN, University of Toronto
NEWTON POLYHEDRA THEORY FOR GENERICLY INCONSISTENT SYSTEMS OF EQUATIONS

Consider a system of equations

$$P_1 = \cdots = P_k = 0$$

in $(\mathbb{C}^*)^n$, where $P_1, \ldots, P_k$ are Laurent polynomials with the supports $A_1, ..., A_k \subset \mathbb{Z}^n$. Assume that the generic system with fixed supports $A_1, ..., A_k$ is inconsistent.

Problem. Compute discrete invariants of $X \subset (\mathbb{C}^*)^n$ defined by a system of equations which is generic **in the set of consistent systems** with supports $A_1, \ldots, A_k$.

I will show how to solve this problem by reducing it to the theory of Newton polyhedra. Unlike the classical situation, not only the Newton polyhedra of $P_1, \ldots, P_k$, but also the supports $A_1, \ldots, A_k$ themselves are relevant. That is, it is not enough to consider only the convex hulls.

SAM PAYNE, Yale
Top weight cohomology of moduli spaces of curves

The top weight cohomology of the moduli space of smooth curves with marked points can be computed (with a degree shift) as the reduced rational homology of a moduli space of stable tropical curves. I will present new applications of this tropical approach, based on recent joint work with M. Chan and S. Galatius.

SANDRA DI ROCCO, KTH, Royal Institute of Technology, Stockholm
Resurgence, Waldschmidt constants and Negative Curves

The resurgence of a homogeneous ideal of points in projective plane is an invariant defined by Bocci and Harbourne in order to measure the relationship between ordinary powers and symbolic powers of the ideal. Resurgence is related to the so called Waldschmidt constant, bounding the order of vanishing of homogeneous forms throw the given points. The study of negative curves on the blow up surface turns out to be an affective tool to compute such invariants. We will present recent results regarding the Klein and Wiman configuration of lines in projective space. This is joint work with Thomas Bauer, Brian Harbourne, Jack Huizenga, Alexandra Seceleanu, and Tomasz Szemberg.

KRISTIN SHAW, Fields Institute
Non-existence of torically maximal hypersurfaces

Simple Harnack curves are extremal objects in real algebraic geometry that were introduced by Mikhalkin. Since then they have appeared in different areas of mathematics and finding their higher dimensional analogues has been an interesting open problem. One proposed generalisation are torically maximal subvarieties. These are real subvarieties of the complex torus whose logarithmic Gauß map is generically totally real. In this talk we will explain why, beyond the case of curves, the only torically maximal projective hypersurfaces are hyperplanes. In higher dimensions we also show that the only real hypersurfaces having a totally real logarithmic Gauß map are hyperplanes of projective spaces. In higher codimension, products of torically maximal hypersurfaces are also torically maximal, but the existence of other examples remains an open problem.

This talk is based on joint work with Erwan Brugallé, Grigory Mikhalkin, and Jean-Jacques Risler.

GREG SMITH, Queen's University
Better Locally-Free Resolutions

Syzygies capture subtle geometric properties of a subvariety in projective space. However, when the ambient space is a product of projective spaces or a general smooth toric variety, minimal free resolutions over the Cox ring are too long and contain many geometrically superfluous summands. After illustrating this problem, we will construct some shorter free complexes that better encode the intrinsic geometry. This talk is based on joint work with Daniel Erman and Christine Berkesch Zamaere.

NICOLA TARASCA, Fields Institute
Du Val curves and the pointed Brill-Noether theorem

The pointed Brill-Noether theorem describes under which condition a general pointed curve admits a linear series with prescribed vanishing sequence at the marked point. While the statement holds for a general pointed curve, no examples was known of *smooth* pointed curves satisfying the theorem. In recent joint work with Gavril Farkas, we show that a general pointed Du Val curve satisfies the theorem. In particular, we give explicit examples of smooth pointed curves of arbitrary genus defined over Q which satisfy the pointed Brill-Noether theorem.

MARTIN ULIRSCH, Fields Institute for Research in Mathematical Sciences
Tropical and non-Archimedean geometry of toric stacks – with a view towards twisted Losev-Manin spaces

In this talk I am going to report on joint work-in-progress with Steffen Marcus and Matthew Satriano concerning the tropical and non-Archimedean geometry of toric stacks. Extending the class of toric varieties, toric stacks are a rather restrictive type of algebraic stack whose geometry can be completely described in terms of a combinatorial object, a so called *stacky fan*. The first part of this talk will be concerned with a reinterpretation of these stacky fans as geometric stacks over the category of rational polyhedral fans (with torsion). Using this language we can then describe two different geometric realizations of these stacky fans as topological stacks, both of which arise naturally as a stack-theoretic non-Archimedean skeleton of the original toric stack. The second half of the talk will deal with a particular example: the moduli space $\mathcal{L}_n^{\leq N}$ of twisted stable chains of projective lines with $n+2$ marked points, where the orders of the stabilizer groups are bounded by N, a so called *twisted Losev-Manin space*. We will show that this moduli space is a root stack over the toric variety of the permutohedron and exhibit its tropicalization as a moduli stack of twisted stable rational tropical chain curves.

ROBERT WILLIAMS, Texas A&M University
Minkowski sums of algebraic varieties

The Minkowski sum is a classical operation, and the sum of two polytopes is a well-known construction. This talk will focus on applying the Minkowski sum to a more geometrically diverse class of objects- algebraic varieties. Unlike with convex bodies, the sum of two algebraic varieties is not necessarily a variety. We will explore the conditions under which Minkowski sum respects Zariski closure as well as when it is well behaved with respect to the dimension and degree of the varieties.

ALEX WOO, University of Idaho
Interval pattern avoidance for K-orbit closures

Let $G = GL(n)$, B the subgroup of upper-triangular matrices, and $K = GL(p) \times GL(q)$ where $p + q = n$. The group K acts with finitely many orbits on the flag variety G/B, and one can study the closures of K-orbits just as one studies Schubert varieties, which are the closures of B-orbits. The set of K-orbits is parameterized by combinatorial objects known as (p, q)-clans. I will explain an older theorem relating interval pattern avoidance on permutations and singularities of Schubert varieties and how to extend this relationship to (p, q)-clans and K-orbit closures.

This is joint work with Ben Wyser and Alexander Yong.