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Symplectic invariants of focus-focus singularities and bifurcations of integrable systems

[joint in progress with Daniele Sepe] Since Duistermaat (1980) it has been known that focus-focus singularities obstruct an integrable system from being toric; these singularities introduce a global topological obstruction known as monodromy. More recently, Vu Ngoc (2002) showed that the presence of focus-focus singularities in integrable systems introduces more than just topological monodromy: for each focus-focus fibre there is an invariant that classifies the germ of the Lagrangian foliation up to symplectomorphism.

At the same time, one can often deform an integrable system so that the focus-focus singularity bifurcates into an elliptic singularity, and elliptic singularities have no such invariants. The obvious question is therefore: what happens to Vu Ngoc's symplectic invariants as the system bifurcates?

As a first step towards answering this question, we have undertaken the computation of symplectic invariants in the Hamiltonian-Hopf bifurcation. The Hamiltonian-Hopf bifurcation is a family of integrable systems on \mathbb{R}^4 that interpolates between the harmonic oscillator and the particle in a champagne bottle, introduced by Bates, which was one of the first systems known to possess focus-focus singularities. In this talk I hope to share some preliminary results and ideas they inspire.