Let $\omega$ denote the arithmetic function that counts the number of distinct prime factors of an integer, and let $d \geq 1$. We develop a bivariate probabilistic model to study the joint distribution of the deterministic vectors $(\omega(n), \omega(n + d))$ with $n \leq x$ as $x \to \infty$, where $n$ and $n + d$ are both required to be squarefree. The following three results are applications:

i) We establish a quantitative version of the bivariate Erdős-Kac theorem, a result first proven in a non-quantitative way by W. LeVeque, on a proper subset of $\mathbb{N}$.

ii) We give two partial results in the direction of a conjecture of Chowla on binary correlations of the Möbius function. Let $\mu_z(n) := \mu^2(n)(-1)^{\omega(n; z)}$, where $\omega(n; z)$ is the number of distinct primes $p | n$ with $p \leq z$. Then as long as $\frac{\log z}{\log x} \to \infty$ as $x \to \infty$, $\sum_{n \leq x} \mu_z(n)\mu_z(n + 1) = o(x)$.

In a related way, we show that if $\mu(n; u) := \mu^2(n)e^{iu\omega(n)}$ then provided that $u = o(1)$ and $u \sqrt{\log_2 x} \to \infty$ as $x \to \infty$ then $\sum_{n \leq x} \mu(n; u)\mu(n + 1; u) = o(x)$.

iii) We finally prove a partial result in the direction of a conjecture of Erdős, Pomerance and Sarközy on the order of magnitude of the number of $n \leq x$ such that $\tau(n) = \tau(n + 1)$.