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The distribution of positive and negative values of Hardy's Z-function

We investigate the distribution of positive and negative values of Hardy's function

$$Z(t) = \zeta(\tfrac{1}{2} + it)\chi(\tfrac{1}{2} + it)^{-1/2},$$

where $\chi(s)$ is the factor from the functional equation for the zeta function,

$$\zeta(s) = \chi(s)\zeta(1 - s).$$

We show that as $T \to \infty$,

$$\mu(I_+(T,T)) \gg T \qquad \text{and} \qquad \mu(I_-(T,T)) \gg T,$$

where $\mu(\cdot)$ denotes Lebesgue measure and

$$\begin{aligned} I_+(T,H) &= \{T < t \le T + H \,:\, Z(t) > 0\}, \\ I_-(T,H) &= \{T < t \le T + H \,:\, Z(t) < 0\}. \end{aligned}$$

We also show that if the Riemann hypothesis and pair correlation conjecture are true, then

$$\mu(I_+(0,T)) \ge .32909\,T \qquad \text{and} \qquad \mu(I_-(0,T)) \ge .32909\,T.$$

This is joint work with A. Ivic.