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Continued Logarithms

Let $1 \leq \alpha \in \mathbb{R}$. Let $y_0 = \alpha$ and recursively define $a_n = \lfloor \log_2 y_n \rfloor$. If $y_n - 2^{a_n} = 0$, then terminate. Otherwise, set

$$y_{n+1} = \frac{2^{a_n}}{y_n - 2^{a_n}}$$

and recurse. This produces the binary (base 2) continued logarithm for y_0:

$$y_0 = 2^{a_0} + \cfrac{2^{a_0}}{2^{a_1} + \cfrac{2^{a_1}}{2^{a_2} + \cfrac{2^{a_2}}{2^{a_3} + \cdots}}}$$

These binary continued logarithms were introduced explicitly by Gosper in his appendix on Continued Fraction Arithmetic. These were further studied by Borwein et. al. extending classical continued fraction recurrences for binary continued logs and investigating the distribution of aperiodic binary continued logarithm terms for quadratic irrationalities – such as can not occur for simple continued fractions.

In this talk we will talk about recent work generalizing many of these results to higher bases.