Measure-Valued Diffusions
Diffusions à valeurs mesurées
(Org: **Xiaowen Zhou** (Concordia))

DONALD DAWSON, Carleton University
Interacting diffusions and their measure-valued limits in an ultrametric setting

Spatial stochastic models and their behaviour in different space and time scales have been intensively studied in Euclidean spaces for many years. In this lecture we review some developments in the study of the analogous questions in a class of ultrametric spaces and the relations between these two settings. We consider interacting $\mathbb{R}_+^k$-valued diffusions for $k = 1, 2$ and the classification of their scaling behaviours based on the degree of transience-recurrence for the associated random walks. In particular we discuss the Fisher-Wright, Feller continuous state branching, catalytic, mutually catalytic branching and self-catalytic branching universality classes and their measure-valued limits. This is based on joint research with Andreas Greven.

SHUI FENG, McMaster University
An Infinite Dimensional Diffusion Associated With Dirichlet Process

The focus of this talk is an infinite dimensional diffusion constructed through coupling. The reversible measure of the process is the Dirichlet process on a countable space. The spectral gap of the process is explicitly identified, and the Poincaré inequality is established. This is a joint work with Laurent Miclo and Feng-Yu Wang.

ROBERT KNOBLOCH, Saarland University
Applications of random characteristics in the context of fragmentation processes

This talk is concerned with a strong law of large numbers for $(Z_\eta^\phi)_{\eta\in(0,1]}$, the so-called *process counted with a random characteristic* ϕ, in the context of fragmentation processes. In this setting random characteristics are stochastic processes that are used to describe certain properties of the block structure of a fragmentation process. For a large class of random characteristics ϕ we prove almost sure convergence and L^1-convergence of Z_η^ϕ as $\eta \downarrow 0$.

Motivated by a problem concerning the energy cost arising in the mining industry Bertoin and Martínez proved in [1] the L^1-convergence for some functional that can be considered as a process counted with a particular characteristic ϕ. In this talk we present a limit theorem for more general random characteristics ϕ. In particular, this enables us to extend the L^1-convergence result obtained in [1] to almost sure convergence in a more general setting. Moreover, in the spirit of [2] and related laws of large numbers for superdiffusions we also prove the almost sure convergence of random empirical measures associated with the stopping line that corresponds to the first blocks, in their respective "line of descent", of size less than $\eta \in (0, 1]$ in a fragmentation process.

References:

[1] J. Bertoin, S. Martínez. Fragmentation energy, *Adv. Appl. Prob.*, **37**, 553–570, 2005

[2] S. C. Harris, R. Knobloch, A. E. Kyprianou. Strong law of large numbers for fragmentation processes, *Ann. Inst. H. Poincaré Probab. Statist.*, **46** (1), 119–134, 2010

EYAL NEUMAN, University of Rochester
Discrete SIR Epidemic Processes and their Relation to Extreme Values of Branching Random Walk

We study the behavior of spatial SIR epidemic models in dimensions two and three. In these models, populations of size N are located at sites of the d-dimensional lattice, and infections occur between individuals at the same or at neighboring sites with infection probability p_N. Susceptible individuals, once infected, remain contagious for one unit of time and then recover, after which they are immune to further infection. We answer the question which was raised in Lalley, Perkins and Zheng (2014) and prove that there exist critical values $p_c(N) > 0$ such that for N large enough, if $p_N > p_c(N)$, then the epidemic survives forever with positive probability. When $p_N < p_c(N)$ we prove that the epidemic dies out in finite time with probability 1.

The behavior of extreme values of branching random walk is a key ingredient in the proof of phase transitions. In this context, we prove that the support of the local time of supercritical branching random walk near criticality, grows in a linear speed. Finally the tail behavior for the right most position reached by subcritical branching random walk is derived.

This is joint work with Xinghua Zheng.

TOM SALISBURY, York University

X-harmonic functions for super-Brownian motion

Dynkin's formalism of X-harmonic functions gives a natural setting in which to discuss super-Brownian motion conditioned on its exit measure. The theory of such functions is still rudimentary, however. We will discuss some joint work with Deniz Sezer (Calgary), that resolve some general questions about X-harmonic functions.

YOUZHOU ZHOU, Zhongnan University of Economics and Law

Asymptotic Behaviour of an Infinitely-Many-Alleles Model with Symmetric Overdominance

In this talk, we consider the limiting distributions of $\pi_{\lambda,\theta}$, the stationary distribution of infinitely-many-alleles diffusion with symmetric overdominance [1]. In [2] the large deviation principle for $\pi_{\lambda,\theta}$ indicates that there are countably many phase transitions for the limiting distribution of $\pi_{\lambda,\theta}$, and the critical points are $\lambda = k(k+1), k \geq 1$. The asymptotic behaviours at critical points , however, are unclear. We will provide a definite description of the critical cases.

References

[1] Ethier,S.N. and Kurtz,T.G.(1998).Coupling and ergodic theorems for Fleming-Viot processes.Ann.Probab,26(2),533-561.
[2] Feng,S.(2009). Poisson-Dirihclet distributions with small mutation rate. Stochastic Process.Appl.,119(6),2082-2094.