Geometric Spectral Theory
Théorie géométrie spectrale
(Org: **Alexandre Girouard** (Laval))

YANNICK BONTHONNEAU, CIRGET - CRM
Resonances in strips: cusp manifolds

I will explain why for generic, negatively curved metrics on cusp manifolds, most of the resonances are contained in a strip at high frequency.

The main step in the argument is to build a semi-classical parametrix for the scattering determinant. This generalizes a theorem of Selberg.

YAIZA CANZANI, Harvard University
Geometry and topology of zero sets of random waves on a manifold

There are several questions about the zero set of Laplace eigenfunctions that have proved to be extremely hard to deal with and remain unsolved. Among these are the study of the size of the zero set, the study of the number of connected components, and the study of the topology of such components. A natural approach is to randomize the problem and ask the same questions for the zero sets of random linear combinations of eigenfunctions. In this talk I will present some recent results in this direction.

BRODERICK CAUSLEY, McGill
Generalized Lawson tau-surfaces and nonmaximality

Recently Penskoi generalized the well-known two-parametric family of Lawson tau-surfaces $\tau_{r,m}$ minimally immersed in spheres to a three-parametric family $T_{a,b,c}$ of tori and Klein bottles minimally immersed in spheres. It was remarked that this family includes surfaces carrying all extremal metrics for the first non-trivial eigenvalue of the Laplace-Beltrami operator on the torus and on the Klein bottle: the Clifford torus, the equilateral torus and the bipolar Lawson Klein bottle $\tilde{\tau}_{3,1}$.

In my talk, I will survey and describe recent progress on this three-parametric family (and subfamilies) of tori and Klein bottles. I will also discuss nonmaximality of metrics induced on $T_{a,b,c}$.

MERC CHASMAN, University of Minnesota Morris
The Clamped Plate in Gauss Space

We consider the analogue in Gauss space of Lord Rayleigh's conjecture for the clamped plate, showing that the first eigenvalue of the bi-Hermite operator on certain domains is bounded below by a constant C_V times the corresponding eigenvalue of a half-space with the same Gaussian measure V. The proof uses rearrangement methods similar to Talenti, Ashbaugh, and Benguria's for the Euclidean clamped plate. We also obtain the numerical bound $C_V \geq 0.91$ by solving an associated minimization problem in terms of parabolic cylinder functions. (Joint work with J. Langford.)

AHMAD EL SOUFI, Francois Rabelais University, Tours
Eigenvalues of the Laplace operator with weights

On a compact Riemannian manifold (M, g), possibly with boundary, we consider the eigenvalues of the weighted Dirichlet energy $\int_M |\nabla u|^2 \sigma\, v_g$ with respect to a weighted L^2 inner product $\int_M u^2 \rho\, v_g$, and discuss the behavior of these eigenvalues when the weights σ and ρ vary within the set of positive functions whose integral over M is fixed. This general context includes several known situations such as Witten Laplacians ($\sigma = \rho$), nonhomogeneous vibrating membranes ($\sigma = 1$), optimal conductivity ($\rho = 1$), etc.

EVANS HARRELL, Georgia Tech
Agmon metrics, exponential location, and the shape of quantum graphs

We show that the Agmon method for establishing exponential decrease of eigensolutions (or subsolutions) can be adapted to quantum graphs. As a generic matter, the rate of decay is controlled by an Agmon metric related to the classical Liouville-Geen estimate for the line, but more rapid decay is typical, arising from the geometry of the graph. We provide additional theorems capturing this effect with alternative Agmon metrics, one adapted to a path and the other using averaging. This is joint work with Anna Maltsev of the University of Bristol, http://arxiv.org/abs/1508.06922

MIKHAIL KARPUKHIN, McGill University
Upper bounds for the first eigenvalue of the Laplacian on non-orientable surfaces

In 1980 Yang and Yau proved the celebrated upper bound for the first eigenvalue on an orientable surface of genus γ. Later Li and Yau gave a simple proof of this bound by introducing the concept of conformal volume of a Riemannian manifold. In the same paper they proposed an approach for obtaining a similar estimate for non-orientable surfaces. We will discuss a formalisation of this approach which also leads to improved bounds.

DIMITRI KOROTKIN, Concordia University
Symplectic geometry of the moduli space of projective structures

We introduce a natural symplectic structure on the moduli space of quadratic differentials with simple zeros and describe its Darboux coordinate systems in terms of so-called homological coordinates. We then show that this structure coincides with the canonical Poisson structure on the cotangent bundle of the moduli space of Riemann surfaces, and therefore the homological coordinates provide a new system of Darboux coordinates. We define a natural family of commuting "homological flows" on the moduli space of quadratic differentials and find the corresponding action-angle variables. The space of projective structures over the moduli space can be identified with the cotangent bundle upon selection of a reference projective connection that varies holomorphically and thus can be naturally endowed with a symplectic structure. Different choices of projective connections of this kind (Bergman, Schottky, Wirtinger) give rise to equivalent symplectic structures on the space of projective connections but different symplectic polarizations: the corresponding generating functions are found. We also study the monodromy representation of the Schwarzian equation associated with a projective connection, and we show that the natural symplectic structure on the the space of projective connections induces the Goldman Poisson structure on the character variety. Combined with results of Kawai, this result shows the symplectic equivalence between the embeddings of the cotangent bundle into the space of projective structures given by the Bers and Bergman projective connections. Talk is based on joint work with M.Bertola and C.Norton.

JEAN LAGACÉ, Université de Montréal
Optimal bounds on a generalised Gauss circle problem

In this joint work with Leonid Parnovski, we study of the integrated density of states of the free periodic Schrödinger operator on $\mathbb{T}^n \times \mathbb{R}^k$ leads to a generalisation of the Gauss circle problem. Geometrically, this generalisation corresponds to computing the area of hyperplanes intersecting a ball of large radius. Using the Poisson summation formula, one can recover upper bounds for the error term just as in the classical case, and they become more precise as both the dimension of the ball and of the hyperspace grows.

What is surprising, is that when the codimension of the hyperplanes is sufficiently small the upper bound on the error term stops improving. Furthermore, we can show that this upper bound is optimal in an average sense. I will show how this behavior happens, and where it leads in the study of periodic Schrödinger operators on $\mathbb{T}^n \times \mathbb{R}^k$.

JEFFREY LANGFORD, Bucknell University
Neumann Comparison Results in Cylindrical Domains

In this talk we will use the the star-function method of Baernstein to obtain Neumann comparison results in cylindrical domains. We begin with a bit of history of comparison results, starting with the classical (Dirichlet) results of Talenti. Our main result compares the solutions of two PDEs, one with given initial data, and one where the data has been Steiner symmetrized in one direction. We show that the solution of the symmetric problem exhibits larger convex means, among other things. We also discuss physical applications and applications to the hot spots problem.

IOSIF POLTEROVICH, Université de Montréal
Nodal geometry of Steklov eigenfunctions

The talk will focus on the geometry of the nodal sets of eigenfunctions of the Steklov eigenvalue problem on Riemannian manifolds. Some recent results and open questions will be discussed.

RAPHAËL PONGE, Seoul National University
Spectral Theory and Conformal Geometry

This talk will have two parts. In the first part, I will a version in conformal geometry of the inequality eigenvalue of Vafa-Witten for Dirac operators. I will also present a reformulation of the Atiyah-Singer index formula for Dirac operators that takes into account the action of the group of conformal diffeomorphisms. This leads us to a whole new family of conformal invariants that are not of the type of the conformal invariants considered by Spyros Alexakis in his proof of the Deser-Swimmer conjecture. In the second part, I present the construction of conformal invariants from nodal sets of conformal invariant operators (e.g., Yamabe and Paneitz operators) and describe applications to curvature prescription problems. The first part is joint work with Hang Wang. The second part is joint work with Yaiza Canzani, Rod Gover, Dmitry Jakobson and Andrea Malchiodi.

BARTEK SIUDEJA, University of Oregon
On mixed Dirichlet-Neumann eigenvalues of triangles.

We consider a fixed triangular domain and various mixed Dirichlet-Neumann eigenvalue problems on that domain. We are interested in the dependence of the smallest eigenvalue of the problem on the choice of the sides for the Dirichlet boundary. It turns out that the longer the Dirichlet side the higher the eigenvalue. Similarly with two Dirichlet sides.

CRAIG SUTTON, Dartmouth College
On hearing the length spectrum of a compact Lie group

It is a long-standing folk-conjecture that the length spectrum of a compact Riemannian manifold is encoded in its Laplace spectrum. This conjecture is known to hold for sufficiently "bumpy" Riemannian manifolds; however, little is known when the manifold possess a "large" isometry group. We demonstrate that for a generic bi-invariant metric on a compact Lie group the singular support of the trace of its associated wave group coincides with the length spectrum of the metric. Consequently, the length spectrum of a generic bi-invariant metric can be recovered from its Laplace spectrum. We also exhibit a substantial collection $\mathcal{G}$ of compact Lie groups U having the property that the conjecture holds for every bi-invariant metric carried by U. Finally, in all the cases considered above, we find that the spectrum of the bi-invariant metric also encodes the rank of its underlying compact Lie group.

STEVE ZELDITCH, Northwestern University
Logarithmic lower bound for the number of nodal domains

In prior work with Junehyuk Jung, we proved that the number of nodal domains tends to infinity along a subsequence of density one of eigenfunctions of the Laplacian on real Riemann surfaces of negative curvature. This talk improves the result by giving a log lower bound. It stems from log scale quantum ergodicity results of Hezari-Riviere and Han.