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Moduli spaces of nonnegatively curved Riemannian metrics

A fundamental problem in Riemannian geometry is to understand which manifolds admit metrics displaying certain types of curvature characteristics. Of particular importance amongst these characteristics are the various sign-based conditions, for example negative sectional curvature, positive Ricci curvature and so on. Existence issues for positive scalar curvature metrics are reasonably well understood, but the situation for positive Ricci and positive or nonnegative sectional curvature metrics is somewhat less clear. The theory of manifolds with negative sectional curvature is well-developed, however the existence question is far from resolved.

For the most part this existence question has been a primary focus of research. However, there is an equally intriguing secondary question. If a manifold admits a given type of metric, how are such metrics distributed among all possible Riemannian metrics on this object? For example are they rare or common? How 'many' metrics and geometries does a given manifold allow for?

To answer these questions, one usually looks at the space of metrics satisfying various given curvature conditions on the manifold, or its quotient by the group of diffeomorphisms, the so-called moduli space of metrics, and studies its respective properties.

In my talk, I will survey and describe recent progress on these questions, focusing primarily on connectedness properties of moduli spaces of nonnegative sectional curvature metrics.