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Multi- to one-dimensional transportation

We consider the Monge-Kantorovich problem of transporting a probability density on $\mathbf{R}^m$ to another on the line, so as to optimize a given cost function. We introduce a nestedness criterion relating the cost to the densities, under which it becomes possible to uniquely solve this problem, by constructing an optimal map one level set at a time. This map is continuous if the target density has connected support. We use level-set dynamics to develop and quantify a local regularity theory for this map and the Kantorovich potentials solving the dual linear program. We identify obstructions to global regularity through examples.