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Chaos and quasiperiodicity

Periodicity, quasiperiodicity, and chaos are the types of typically observed in general dynamical systems. The Birkhoff Ergodic Theorem asserts that the Birkhoff time average, $\Sigma_{n=0}^{N-1} f(x_n)/N$ of a function f along a length N ergodic trajectory (x_n) of a function T converges to the space average $\int f d\mu$, where μ is the unique invariant probability measure for T. This relationship between the time and space averages is powerful, since often a time series is the only information available. However, the convergence of the Birkhoff average is slow, with an error of order N^{-1} for a length N trajectory. We present a modified Birkhoff average technique by giving very small weights to the terms to $f(x_n)$ when n is near 0 or $N-1$. Our method is to calculate $\Sigma_{n=0}^{N-1} w(n/N) f(x_n)$, where the weighting function w vanishes smoothly at the ends 0 and 1. This method is a significant improvement: when (x_n) is a trajectory on a quasiperiodic torus and f and T are infinitely-many times differentiable, our method of weighted Birkhoff average converges exponentially fast to $\int f d\mu$ with respect to the number of iterates N, *i.e.* with error decaying faster than N^{-m} for every integer m. As a result of this speed, we are able to obtain high precision values for $\int f d\mu$ with relatively low computational cost. Our weighted Birkhoff average is a powerful computational tool for computing rotation numbers and conjugacies. This is joint work with Suddhasattwa Das, Yoshitaka Saiki and James Yorke.