Complex Analysis and Operator Theory
Analyse complexe et théot théorie des opérateurs
(Org: **Javad Mashreghi** and/et **Thomas Ransford** (Laval))

MAXIME FORTIER BOURQUE, University of Toronto
The holomorphic couch theorem

The only obstructions to moving a holomorphic couch in a holomorphic house are topological. More precisely, if two conformal embeddings between Riemann surfaces are homotopic, then they are homotopic through conformal embeddings. In other words, the space of all conformal embeddings in a given homotopy class is path-connected. The proof of this theorem uses several tools from Teichmüller theory such as quasiconformal maps, quadratic differentials, and extremal length.

MAXIM BURKE, University of Prince Edward Island
Approximation and interpolation by entire functions with restriction of the values of the derivatives

A theorem of Hoischen states that given a positive continuous function $\varepsilon : \mathbb{R}^n \to \mathbb{R}$, an unbounded sequence $0 \le c_1 \le c_2 \le \dots$ and a closed discrete set $T \subseteq \mathbb{R}^n$, any C^∞ function $g : \mathbb{R}^n \to \mathbb{R}$ can be approximated by an entire function f so that for $k = 0, 1, 2, \dots$, for all $x \in \mathbb{R}^n$ such that $|x| \ge c_k$, and for each multi-index α such that $|\alpha| \le k$,

(a) $|(D^\alpha f)(x) - (D^\alpha g)(x)| < \varepsilon(x)$;

(b) $(D^\alpha f)(x) = (D^\alpha g)(x)$ if $x \in T$.

We show that if $C \subseteq \mathbb{R}^{n+1}$ is meager, $A \subseteq \mathbb{R}^n$ is countable and for each multi-index α and $p \in A$ we are given a countable dense set $A_{p,\alpha} \subseteq \mathbb{R}$, then we can require also that

(c) $(D^\alpha f)(p) \in A_{p,\alpha}$ for $p \in A$ and α any multi-index;

(d) if $x \notin T$, $q = (D^\alpha f)(x)$ and there are values of $p \in A$ arbitrarily close to x for which $q \in A_{p,\alpha}$, then there are values of $p \in A$ arbitrarily close to x for which $q = (D^\alpha f)(p)$;

(e) for each α, $\{x \in \mathbb{R}^n : (x, (D^\alpha f)(x)) \in C\}$ is meager in $\mathbb{R}^n$.

Clause (d) is a surjectivity property which can be strengthened to allow for finding solutions in A to equations of the form $q = h^*(x, (D^\alpha f)(x))$ under similar assumptions, where $h(x, y) = (x, h^*(x, y))$ is one of countably many given fiber-preserving homeomorphisms of open subsets of $\mathbb{R}^{n+1} \cong \mathbb{R}^n \times \mathbb{R}$.

We also prove a weaker corresponding result with "meager" replaced by "Lebesgue null." In this context, the approximating function is C^∞ rather than entire, and we do not know whether it can be taken to be entire.

RICHARD FOURNIER, Dawson College and CRM
An extension of Jack's lemma

After a short review of existing literature, we shall prove an extension of the (in)famous Jack's lemma based on ideas due mainly to Ruscheweyh and Sheil-Small concerning bound-preserving convolutions operators on classes of polynomials. This is partly joint work with Marius Serban.

PAUL GAUTHIER, Université de Montréal
Approximation by: Riemann zeta-function; polynomials (rational functions) with prescribed zeros (poles)

Andersson showed that an improvement of the spectacular theorem of Voronin on the universality of the Riemann zeta-function is equivalent to a natural problem on polynomial approximation with prescribed zeros. Classical approximation is with respect to rational functions with prescribed poles. We consider meromorphic approximation with prescribed poles on Riemann surfaces, bearing in mind that poles and zeros have a similar nature for meromorphic functions. Any new results are jointly with Fatemeh Sharifi.

KARIM KELLAY, Université de Bordeaux
Echantillonnage et interpolation multiple dans l'espace de Fock

Nous étudions les ensembles d'interpolation, d'unicité et d'échantillonnage multiple pour les espaces de Fock classiques dans le cas où la multiplicité est non bornée. Nous montrons, dans le cas hilbertien ainsi que celui de la norme uniforme, qu'il n'y a pas de suites simultanément d'échantillonnage et d'interpolation lorsque la multiplicité tend vers l'infini. Ceci répond partiellement à une question posée par Brekke et Seip. Travail conjoint avec A. Borichev, A.Hartmann et X.Massaneda

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JAVAD RASTEGARI, The University of Western Ontario
Fourier inequalities in weighted Lebesgue and Lorentz spaces

The Fourier transform is an operator of strong type $(1,\infty)$ and $(2,2)$. By interpolation one obtains the Hausdorff-Young inequality which implies that the Fourier transform maps L^p into $L^{p'}$ when $1 \leq p \leq 2$. A generalization of this inequality to weighted L^p spaces with power weights is known as Pitt's inequality. However, a still open problem is to characterize those weight functions u, w for which the Fourier transform maps $L^p(w)$ into $L^q(u)$.

The Lorentz spaces, $\Lambda^p(v)$, provide a powerful tool to obtain Fourier inequalities in weighted Lebesgue spaces. Our focus is on the Fourier series in weighted Lorentz spaces. We provide relations between weight functions and exponents, that are necessary and sufficient for the boundedness of the Fourier coefficients, viewed as a map between Lorentz spaces. We also apply our results to weighted Lebesgue spaces, $L \log L$, and Lorentz-Zygmund spaces.

This is joint work with Gord Sinnamon.

ERIC SCHIPPERS, University of Manitoba
The Faber isomorphism, Grunsky operator, and Schiffer operator on quasidisks

In previous work we characterized the boundary values of harmonic functions of finite Dirichlet energy on a quasidisk as a certain Besov space on the quasicircle. We also showed that the Dirichlet problem and Plemelj-Sokhotski jump problems have solutions which depend continuously on the boundary data. In this talk we use these results to define a reflection on harmonic functions in quasidisks obtained by first taking the boundary values and then solving the Dirichlet problem on the complementary domain. We derive formulas relating Faber series, the Grunsky operator and this reflection. In particular, we give an isomorphism between Dirichlet spaces of quasidisks closely related to Faber series, which respects composition of the conformal map between the complements. Finally, we show that the space of L^2 harmonic one forms on a quasidisk is isomorphic to the direct sum of the Bergman spaces of the quasidisk and its complement, and relate this to an operator of Schiffer. Joint work with Wolfgang Staubach.

FATEMEH SHARIFI, University of Western Ontario
zero-free approximation

Fatemeh Sharifi, University of Western ontario,

Title: Zero free approximation.

Abstract. Let E be a compact subset of the complex plane with connected complement. We define $A(E)$ to be the class of all complex continuous functions on E that are holomorphic in the interior E^0 of E. The remarkable theorem of Mergelyan states that every $f \in A(E)$ is uniformly approximable by polynomials on E, but is it possible to realize such an approximation by polynomials that are zero-free on E? This question was proposed (but not pubished) by P. Gauthier and subsequently posed independently (and published) by J. Andersson. Recently, Arthur Danielyan described a class of functions for which zero-free approximation is possible on an arbitrary E. I intend to present a generalization of his work on Riemann surfaces. This is joint work with Paul Gauthier.

ANUSH STEPANYAN, Université Laval
Nonlinear maps preserving the minimum and surjectivity moduli

Let X and Y be infinite-dimensional complex Banach spaces, and let $\mathcal{B}(X)$ (resp. $\mathcal{B}(Y)$) denote the algebra of all bounded linear operators on X (resp. on Y). We describe surjective maps φ from $\mathcal{B}(X)$ to $\mathcal{B}(Y)$ satisfying

$$\mathrm{c}(\varphi(\mathrm{S}) \pm \varphi(\mathrm{T})) = \mathrm{c}(\mathrm{S} \pm \mathrm{T})$$

for all $S,\ T \in \mathcal{B}(X)$, where $\mathrm{c}(\cdot)$ stands either for the minimum modulus, or the surjectivity modulus, or the maximum modulus. We also obtain analog results for the finite-dimensional case.

MALIK YOUNSI, Stony Brook University
Removable sets and Koebe's Conjecture

It was observed by He and Shcramm in the 1990's that conformally removable sets are of fundamental importance in the study of the famous Koebe's Conjecture on the conformal equivalence of arbitrary planar domains with circle domains. More precisely, He and Shcramm posed two conjectures on the relationship between the so-called rigidity of a circle domain and the removability of its boundary. In this talk, we shall present a result on countable unions of conformally removable sets. As an application, we show that the two aforementioned conjectures of He and Schramm are in fact equivalent, at least for a large family of circle domains.