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Approximation and interpolation by entire functions with restriction of the values of the derivatives

A theorem of Hoischen states that given a positive continuous function $\varepsilon : \mathbb{R}^n \to \mathbb{R}$, an unbounded sequence $0 \leq c_1 \leq c_2 \leq \dots$ and a closed discrete set $T \subseteq \mathbb{R}^n$, any C^∞ function $g : \mathbb{R}^n \to \mathbb{R}$ can be approximated by an entire function f so that for $k = 0, 1, 2, \dots$, for all $x \in \mathbb{R}^n$ such that $|x| \geq c_k$, and for each multi-index α such that $|\alpha| \leq k$,

(a) $|(D^\alpha f)(x) - (D^\alpha g)(x)| < \varepsilon(x)$;

(b) $(D^\alpha f)(x) = (D^\alpha g)(x)$ if $x \in T$.

We show that if $C \subseteq \mathbb{R}^{n+1}$ is meager, $A \subseteq \mathbb{R}^n$ is countable and for each multi-index α and $p \in A$ we are given a countable dense set $A_{p,\alpha} \subseteq \mathbb{R}$, then we can require also that

(c) $(D^\alpha f)(p) \in A_{p,\alpha}$ for $p \in A$ and α any multi-index;

(d) if $x \notin T$, $q = (D^\alpha f)(x)$ and there are values of $p \in A$ arbitrarily close to x for which $q \in A_{p,\alpha}$, then there are values of $p \in A$ arbitrarily close to x for which $q = (D^\alpha f)(p)$;

(e) for each α, $\{x \in \mathbb{R}^n : (x, (D^\alpha f)(x)) \in C\}$ is meager in $\mathbb{R}^n$.

Clause (d) is a surjectivity property which can be strengthened to allow for finding solutions in A to equations of the form $q = h^*(x, (D^\alpha f)(x))$ under similar assumptions, where $h(x, y) = (x, h^*(x, y))$ is one of countably many given fiber-preserving homeomorphisms of open subsets of $\mathbb{R}^{n+1} \cong \mathbb{R}^n \times \mathbb{R}$.

We also prove a weaker corresponding result with "meager" replaced by "Lebesgue null." In this context, the approximating function is C^∞ rather than entire, and we do not know whether it can be taken to be entire.