Algebraic Combinatorics
Combinatoire algébrique
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FRANÇOIS BERGERON, UQAM

Positivity for rectangular Catalan combinatorics

We will survey the various positivity conjectures (and results) for the broad generalizations of the shuffle conjecture (theorem).

NANTEL BERGERON, York University

Why find cancelation free formula for antipode?

[Joint work with Carolina Benedetti]

Given a family of combinatorial objects we often have an associated graded Hopf algebra. Such algebraic structures encode the associations and decompositions of the objects we study. The antipode is a map from the Hopf algebra into itself that is defined recursively and is difficult to compute in general. Is it worth it to find a cancelation free formula for it?

We start with the Hopf algebra of graphs and show the cancelation free formula of Humpert and Martin for its antipode. We will see that such formula gives a structural understanding of certain evaluations of the combinatorial invariants for graphs. In particular we recover very nicely a classical theorem of Stanley for the evaluation of the chromatic polynomial at -1.

We then give a general framework where we systematically obtain cancelation free formulas for antipodes. More precisely, we define the notion of strongly linearizable Hopf monoids and show how to get cancelation free formula for antipodes in those cases. This allows us to obtain a cancelation free formula many of the combinatorial Hopf algebras in the literature and more.

FRÉDÉRIC CHAPOTON, CNRS et Université de Strasbourg

From Associahedra to Stokes Polytopes

Associahedra form a classical familly of polytopes, with many connections ranging from homotopy to cluster algebras. They have been generalized in several different ways. This talk will present a generalization due to Baryshnikov, and motivated initially by deformations of quadratic differentials. These polytopes seem to share many of the good properties of associahedra; in particular one can define analogues of the Tamari lattices. The underlying combinatorics is that of quadrangulations, and one can expect close connections with representation theory through exceptional sequences.

MATTHEW DYER, University of Notre Dame

Twisted highest weight modules and shellability

We will describe some aspects of (one variant of) a theory of twisted (thickened) highest weight modules for certain highest weight categories. This has as consequences general analogues (in some cases, stronger versions) of various well known phenomena in Lie theory. For instance, factorization of Shapovalov determinants in this setting is often a consequence of factorizations of Shapovalov matrices. It is possible to give a general construction of all highest weight categories admitting structure of this type, but difficult to understand the finer details of their combinatorics and to prove that natural highest weight categories arising in Lie theory (e.g. attached to Coxeter groups) admit such structure. However, extensive classes of examples are known in which the underlying combinatorics of twisted highest weight modules is especially simple and may be described in terms of classical notions of shellability; for example, shellable pure simplicial complexes may be characterized amongst pure simplicial complexes as exactly those for which certain associated highest weight representation categories admit such structure.

THOMAS MCCONVILLE, MIT

Lattice structure of Grassmann-Tamari orders

The Tamari order is a poset of bracketings whose covering relations are given by a left-to-right associativity law. Many nice properties of the Tamari order may be deduced using a well-studied map from permutations to bracketings. Recently, Santos, Stump, and Welker introduced the Grassmann-Tamari order, a poset of maximal nonkissing collections of paths in a rectangle. Generalizing the map from permutations to bracketings, we will define a map from a family of biclosed sets to the facets of the nonkissing complex. Using this map, we can show that the Grassmann-Tamari order is a congruence-uniform lattice.

OLIVER PECHENIK, University of Illinois at Urbana-Champaign

Puzzles and equivariant K-theory of Grassmannians

The cohomology of the Grassmannian has a basis given by Schubert varieties. The structure coefficients of this ring are the celebrated Littlewood-Richardson coefficients, and are calculated by any of the Littlewood-Richardson rules. This story has been extended to K-theory by A. Buch (2002) and to torus-equivariant cohomology by A. Knutson-T. Tao (2003). It is natural to unify these theories via a combinatorial rule for structure coefficients in equivariant K-theory. In 2005, A. Knutson-R. Vakil used puzzles to conjecture such a rule. Recently we proved the first combinatorial rule for these coefficients. Using our new rule, we construct a counterexample to the Knutson-Vakil conjecture and prove a mild correction to it. (Joint work with Alexander Yong)

VINCENT PILAUD, CNRS & LIX, École Polytechnique

Brick polytopes, lattice quotients, and Hopf algebras

This talk is motivated by the deep connections between the combinatorial properties of permutations, binary trees, and binary sequences. Namely, classical surjections from permutations to binary trees (BST insertion) and from binary trees to binary sequences (canopy) yield:

- lattice morphisms from the weak order, via the Tamari lattice, to the boolean lattice;
- normal fan coarsenings from the permutahedron, via Loday's associahedron, to the parallelepiped generated by the simple roots $\mathbf{e}_{i+1} - \mathbf{e}_i$;
- Hopf algebra inclusions from Malvenuto-Reutenauer's algebra, via Loday-Ronco's algebra, to Solomon's descent algebra.

In this talk, we present an extension of this framework to acyclic k-triangulations of a convex (n+2k)-gon, or equivalently to acyclic pipe dreams for the permutation $(1, \dots, k, n+k, \dots, k+1, n+k+1, \dots, n+2k)$. These objects are in bijection with the classes of the congruence of the weak order on $\mathfrak{S}_n$ defined as the transitive closure of the rewriting rule $UacV_1b_1 \cdots V_kb_kW \equiv^k UcaV_1b_1 \cdots V_kb_kW$, for letters $a < b_1, \dots, b_k < c$ and words $U, V_1, \dots, V_k, W$ on $[n]$. It enables us to transport the known lattice and Hopf algebra structures from the congruence classes of $\equiv^k$ to these acyclic pipe dreams. We will describe the cover relations in this lattice and the product and coproduct of this algebra in terms of pipe dreams. We will also recall the connection to the geometry of the brick polytope.

NATHAN READING, North Carolina State University

Lattice congruences on the weak order

Lattice congruences on the weak order on a finite Coxeter group are most naturally understood in terms of the geometry of the associated arrangement of reflecting hyperplanes. In this talk, I will begin with an overview of lattice congruences, aimed at combinatorialists. I will then describe how the geometric conditions governing lattice congruences on the weak order can be distilled into a local combinatorial rule. Time permitting, I will also give a description of lattice congruences on the weak order on permutations in terms of the combinatorics noncrossing arc diagrams.

SALVATORE STELLA, INdAM - Università degli studi di Roma "La Sapienza"
Kac-Moody groups, double Bruhat cells, and cluster algebras

Kac-Moody groups and cluster algebras, while being fundamentally different objects, share surprisingly many notable features. For example they are both classified in terms of root systems and Dynkin diagrams. Many varieties naturally associated to Kac-Moody groups carry a cluster algebra structure but, in general, its type is not related to the type of the group itself.

Motivated by this clash of types, Yang and Zelevinsky studied a particular class of reduced double Bruhat cells of a Lie group and proved that their rings of coordinates are cluster algebras with principal coefficients of the same type of the underlying group. Moreover, and arguably more interestingly, they were able to give expressions for all the cluster variables in terms of generalized minors.

In this talk I will present an ongoing work, joint with D. Rupel and H. Williams, in which we extend the results by Yang and Zelevinsky to any Kac-Moody group.

STEPHANIE VAN WILLIGENBURG, University of British Columbia
Littlewood-Richardson rules for symmetric skew quasisymmetric Schur functions

Symmetric skew quasisymmetric Schur functions are a generalization of skew Schur functions and contain skew Schur functions as a special case. One way of expanding skew Schur functions in terms of Schur functions is to use the famed version of the classical Littlewood-Richardson rule involving Yamanouchi words. This given, a natural question to consider is whether there exists an analogous rule for symmetric skew quasisymmetric Schur functions.

In this talk we will give two Littlewood-Richardson rules for symmetric skew quasisymmetric Schur functions that are analogous to the aforementioned version of the classical Littlewood-Richardson rule. Furthermore, both our rules have the nice property that they contain the classical version as a special case. We will then apply our rules to classify symmetric skew quasisymmetric Schur functions diagrammatically. This talk is based on joint work with Christine Bessenrodt and Vasu Tewari.

GREG WARRINGTON, University of Vermont
Sweep map combinatorics

I'll discuss some combinatorics connected to the sweep map on lattice paths that arises in the theory of the q,t-Catalan and related objects.

NATHAN WILLIAMS, Université de Québec à Montréal (LaCIM)
Coincidental Types and their Minuscule Doppelgängers

For each coincidental type $X_n \in \{A_n, B_n, H_3, I_2(m)\}$, there exists a minuscule poset that is a "doppelgänger" of the root poset $\Phi^+(X_n)$—both posets have a related number of linear extensions and a related number of plane partitions of height k. Furthermore, there is a second minuscule poset whose top half is $\Phi^+(X_n)$. These two facts are related.

We synthesize M. Haiman's rectification, K. Purbhoo's folding, H. Thomas and A. Yong's minuscule K-theoretic Schubert calculus techniques, and a remark made by R. Proctor to give a framework for combinatorial proofs of these poset coincidences. This is joint work with Zachary Hamaker, Rebecca Patrias, and Oliver Pechenik.

MIKE ZABROCKI, York University
Symmetric group characters as symmetric functions

We define a basis of the symmetric functions which are characters and play the same role for the symmetric groups S_n that the Schur functions play for the general linear groups Gl_n. This basis has as structure coefficients the stable Kronecker coefficients.

This is joint work with Rosa Orellana (Dartmouth College)