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Exploring the Beta Invariant over Toric Varieties

When faced with a line bundle L on a projective algebraic variety X over $\mathbb{C}$, it is useful to be armed with some invariants to understand what we are dealing with. To this end, we consider the blowup at a point whose exceptional divisor is E. Visually, once presented with a shape to each point of which a direction has been assigned, we are inserting a copy of $\mathbb{C}^n$ at a problem point in order to more closely examine the point. These tools suffice to give us a new invariant to assign to our line bundle.

In this talk we follow up on the paper by McKinnon and Roth, exploring the invariant $\beta(x) = \int_0^\infty \frac{Vol(\pi^\star L - \gamma E)}{Vol(L)} d\gamma$; we present an alternative closed form for this invariant for a line bundle L on a toric variety, exploring the asymptotic ability of global sections of a line bundle to approximate it. We briefly discuss the convenience of toric varieties as testing grounds for invariants in algebraic geometry, using this invariant $\beta(x)$ as a launch pad for our exploration. In particular, the visually striking cases of $\mathbb{P}^1 \times \mathbb{P}^1$ and $\mathbb{P}^n$ for a general n will be showcased.

We then make the explicit connection of the ingredients of $\beta(x)$ to volumes of Okounkov bodies of the corresponding line bundle and variety (following earlier work of Okounkov and Khovanskii).