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Hard Lefschetz property for contact manifolds

In the literature, there are two different versions of Hard Lefschetz theorems for a compact Sasakian manifold. The first version, due to Kacimi-Alaoui, asserts that the basic cohomology of a compact Sasakian manifold satisfies the Hard Lefschetz property. The second version, established more recently by Cappelletti-Montano, De Nicola, and Yudin, holds for the usual De Rham cohomology of a compact Sasakian manifold. In this talk, we will discuss a new approach to the Hard Lefschetz property of Sasakian manifolds, using the formalism of odd dimensional symplectic geometry. It leads to a Hard Lefschetz theorem for the more general K-contact manifolds, which immediately implies that the two existing versions of Hard Lefschetz theorem are logically equivalent to each other. Our method sheds new insights on the topology of a Sasakian manifold. For instance, we will discuss how to use it to construct simply-connected K-contact manifolds which do not support any Sasakian structures in any dimension greater than or equal to seven. This in particular answers an open question asked by Boyer and late Galicki. If time permits, we will also discuss recently discovered topological obstructions to the existence of a Sasakian metric.