Nonlinear Evolutionary Equations
Équations d'évolution non linéaires
(Org: **Dong Li** (UBC) and/et **Xinwei Yu** (Alberta))

WENYING FENG, Trent University
Topological methods for nonlinear boundary value problems

In this talk, I will introduce the class of (a, q)-stably solvable maps and their properties. New results for operators in the form LF, where L is a bounded linear operator and F is nonlinear are proved. This class of maps is important in applications as many differential equations can be written as $LF(u) = u$. I will also discuss three different approaches, the (a, q)-stably solvable maps, fixed point index and iterative methods in studying nonlinear boundary value problems.

NING JU, Oklahoma State University
Long time behavior of solutions of Primitive Equations

The Primitive Equations (PEs) are a fundamental set of partial differential equations in geophysics modeling large scale motion of ocean and atmosphere on earth. The global wellposedness theory for strong solutions and existence of global attractor for these solutions were established about a decade ago. In this talk, some new results about the long-time behavior of the solutions of PEs obtained recently by the speaker and cooperator will be presented. These are about finite dimensions of global attractor of strong solutions. H^2 regularity and uniqueness of z-weak solutions will also be discussed if time permits.

XINYANG LU, McGill University
Evolution equations in epitaxial growth

Evolution equations modeling epitaxial growth with elasticity on vicinal surfaces was first proposed by Duport, Politi and Villain, and by Tersoff, Phang, Zhang and Lagally in the discrete case. Continuum models have been studied by E, Xu and Xiang for rigid substrates, and by Tekalin and Spencer for elastic substrates. Such equations are generally 4th order, non-local, non-linear, parabolic equations. In this talk, I will present some new results about existence, uniqueness and regularity of weak/strong solutions of these equations. Joint works with Davoli, Fonseca and Leoni.

MING MEI, Champlain College and McGill University
Stationary Solutions to Hybrid Quantum Hydrodynamical Model of Semiconductors in Bounded Domain

In this talk we consider a hybrid quantum hydrodynamic model (H-QHD) of semiconductors. First of all, taking into account the quantum effects in the semiconductor device, we derive a new model called the hybrid quantum hydrodynamic model coupled with the Poisson equation for electric potential. In particular, we write the Bohm potential in a revised form. This new potential is derived heuristically assuming that the energy of the electrons depends on the charge density n and on ∇n just in a restricted part of the device domain, whereas the remaining parts are modeled classically. Namely, the device is designed as some part with the quantum effects and some part without the quantum effects. The main target is to investigate the existence of the stationary solutions for the hybrid quantum hydrodynamic model. In order to do so, by the Leray-Schauder fixed point theorem, we first show the existence and uniqueness of weak solutions for the case that the quantum effect can be small but never zero. Further we prove that solution to the fully hybrid problem (obtained assuming that the quantum effect may disappeared in a given part of the domain), can be obtained as limit solution of a class of H-QHD equations, for variable, but non zero, quantum effects. Finally, we carry out some numerical simulations for different devices, which also confirm our theoretical results.

This is a joint work with F. Di Michele, B. Rubino and R.Sampalmieri

ALIREZA SAYYIDMOUSAVI, McGill University
Finite-element solution for compressible Navier-Stokes flow with density-dependent viscosity and Navier-slip at the wall

Motivated by comparison with a perturbation solution for weakly compressible force-driven flow through a cylinder of fixed cross-section, with density-dependent viscosity and Navier-slip at the wall, a strategy for the numerical solution of the problem using the Finite Element Method is proposed. The element used in the simulations has five degrees of freedom; pressure, velocity in x, y and z directions and temperature. A parabolic velocity profile is imposed at the inlet in such a way that the relationship between centerline velocity and the wall velocity ensures the satisfaction of the Navier-Slip condition at the inlet. The results are compared with the perturbation solution for both approximate and full Navier-slip conditions.

GANTUMUR TSOGTGEREL, McGill University
Well posedness theory for some regularized models of turbulence

Generalizing the Navier-Stokes-α model, Fried and Gurtin recently introduced the Navier-Stokes-$\alpha\beta$ equations, as a turbulence model with a solid continuum mechanical foundation. An attractive feature of this model is that boundary conditions arise naturally. In this talk, we consider the so-called wall-eddy boundary conditions, a replacement of the no-slip boundary conditions. We will discuss some results on the well posedness of the problem. If time permits, we will also discuss a model of turbulence with degenerate coefficients.

VITALI VOUGALTER, University of Toronto
Existence of stationary solutions for some integro-differential equations with anomalous diffusion

The work deals with the existence of solutions of an integro-differential equation arising in population dynamics in the case of anomalous diffusion involving the negative Laplace operator raised to a certain fractional power. The proof of existence of solutions is based on a fixed point technique. Solvability conditions for non-Fredholm elliptic operators in unbounded domains along with the Sobolev inequality for a fractional Laplacian are being used.

KAZUO YAMAZAKI, Department of Mathematics and Statistics, Washington State University
On stochastic partial differential equations in fluid mechanics

We review some results on the stochastic partial differential equations (SPDE) in fluid mechanics, namely the Navier-Stokes and other closely related systems. In particular, we discuss the global existence of a martingale solution for the 3D micropolar fluid (MPF) and magneto-micropolar fluid (MMPF) systems, 3D nonhomogeneous magnetohydrodynamics system, and 2D Boussinesq system with zero dissipation. Moreover, we discuss the existence and uniqueness of an invariant measure as well as the exponential convergence of trajectories to the unique invariant measure for the 2D magnetic Benard problem, 2D MPF and MMPF systems. If time permits, we also discuss large deviation principle results.

HONG ZHANG, Brown University
Schauder estimate for concave nonlocal parabolic equation

In this talk, I will present some recent results on Schauder estimate for concave nonlocal parabolic equation. In particular, no regularity and symmetry are assumed on the kernel. We extend the classical Evan-Krylov for concave parabolic equations. This is a joint work with Hongjie Dong.

KUN ZHAO, Tulane University, New Orleans, LA ,USA
Uniform Distribution in Negative Chemotaxis

In contrast to random diffusion without orientation, chemotaxis is the biased movement of organisms toward the region that contains higher concentration of beneficial or lower concentration of unfavorable chemicals. The former often refers to the attractive chemotaxis and latter to the repulsive chemotaxis. Chemotaxis has been advocated as a leading mechanism to account for the morphogenesis and self-organization of a variety of biological coherent structures such as aggregates, fruiting bodies, clusters, spirals, spots, rings, labyrinthine patterns and stripes, which have been observed in experiments. In this talk, I will present some recent results regarding the rigorous analysis of a nonlinear PDE model arising from the study of repulsive chemotaxis. In particular, local/global well-posedness, long-time asymptotic behavior and diffusion limits of classical solutions will be discussed. The long-time behavior results show that constant equilibrium states are stable, which indicates that chemo-repulsion problem with logarithmic chemotactic sensitivity exhibits a strong tendency against pattern formation. The diffusion limit results demonstrate that the chemically diffusive model is consistent with the non-diffusive model under certain boundary conditions, which may help reduce the computational cost for numerical simulation of the model.