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Utility maximization in a regime switching model with convex portfolio constraints and margin requirements

We study a problem of stochastic control in mathematical finance, with the goal of maximizing expected utility of investment and consumption over a finite trading horizon. The asset prices are modeled by Ito processes, for which the market parameters are subject to *regime switching* in the sense of being adapted to the joint filtration of the driving Brownian motion and a finite-state Markov chain which models "regime states" of the market. The vector of portfolios is constrained to a specified closed and convex set, and margin payments are levied on the investor, resulting in a wealth equation which is *nonlinear* in the portfolio. We proceed by the method of *conjugate duality* to construct a *dual optimization problem* together with *optimality relations* between putative solutions of the given (i.e. "primal") optimization problem and the dual optimization problem. These optimality relations are then used to address the specific cases of *power-type* and *logarithmic* utility functions, with *convex cone* portfolio constraints, and a higher rate of interest for borrowing than for lending. We get completely explicit optimal portfolios and characterize the optimal consumption rate as the solution of a backward stochastic differential equation (BSDE) "driven" by the canonical martingales of the regime-state Markov chain. For the power utility function this is a rather unconventional BSDE, to which standard existence results do not apply, and accordingly we establish existence and uniqueness of solutions for this BSDE.