Computational and Topological Methods in Dynamical Systems
Calcul et méthodes topologiques en systèmes dynamiques
(Org: **Tomasz Kaczynski** (Sherbrooke) and/et **Jean-Philippe Lessard** (Laval))

MAXIME BREDEN, ENS Cachan - Université Laval
Rigorous computation of maximal local (un)stable manifold patches by the parameterization method

I'll present an automatic procedure for computing and validating high order polynomial expansions of local (un)stable manifolds for equilibria of ordinary differential equations. These invariant manifolds are fundamental blocks granting information about the global dynamic of the system. Our aim is to provide a better understanding of the paramterization method within the framework of rigorous computation, and to make it easier to further use the computed manifolds (for instance to prove existence of connecting orbits). The method on which this work is based was already used to rigorously compute connecting orbits with the help of (un)stable manifolds, but because of the lack of systematic procedure to select the various parameters, tedious trial and errors were required for the proof to succeed. I'll explain how the scaling of the eigenvectors can be used in a flexible way to adapt the computation of the manifold to the problem at hand (a fast-slow system for example), and show how to track the influence of the scaling in validation estimates (like the *radii polynomials*), which allows for a cheap optimization scheme for the scaling. An example of a work using this procedure (to prove the existence of traveling waves for the *suspended bridge equation*) will be presented by Maxime Murray in another talk in this session.

MARCIO GAMEIRO, University of São Paulo at São Carlos
Continuation of Point Clouds via Persistence Diagrams

We present a mathematical and algorithmic framework for the continuation of point clouds via persistence diagrams. We show that the persistence map, which assigns a persistence diagram to a point cloud, is differentiable. This allows us to apply the Newton-Raphson continuation method in this setting. Given an initial point cloud P and a its corresponding persistence diagram PD, we apply continuation to find a new point cloud P' close to P, that have a prescribed persistence diagram PD' close to PD. We present algorithms to perform the continuation as well as some computational results. This is joint work with Yasuaki Hiraoka (WPI-AIMR, Tohoku University) and Ippei Obayashi (WPI-AIMR, Tohoku University).

JASON MIRELES JAMES, Florida Atlantic University
Validated computation of connecting orbits in infinite dimensions

I will discuss some computer assisted arguments which establish the existence of connecting orbits for infinite dimensional dynamical systems. The idea is to study a nonlinear operator describing orbits which begin on the local unstable manifold of one hyperbolic fixed point and terminate on the local stable manifold of another. Good numerics lead to approximate zeros of this operator, and the existence of a true zero is obtained by showing that a related Newton-Like operator is a contraction in a suitable neighborhood of the numerical approximation. A critical point is that validated local analysis of the fixed points, their spectra/eigenspaces, as well as their local stable/unstable manifolds are needed in order to frame the analysis. I will show results involving a model of population dynamics with seasonal spatial dispersion.

TOMASZ KACZYNSKI, Université de Sherbrooke
Combinatorial and classical vector field dynamics

Forman's discrete Morse theory is an analogy of the classical Morse theory with, so far, only informal ties. Our goal is to establish a formal bridge on the level of induced dynamics. Following Forman's 1998 paper on "Combinatorial vector fields and dynamical systems", we start with a possibly non-gradient combinatorial vector field. We construct a flow-like upper semi-continuous acyclic-valued mapping whose dynamics is equivalent to the dynamics of Forman's combinatorial vector field, in the sense that isolated invariant sets and index pairs are in one-to-one correspondence. This is a joint work with M. Mrozek and Th. Wanner.

JEAN-PHILIPPE LESSARD, Université Laval
Rigorous numerics for ill-posed PDEs: periodic orbits in the Boussinesq equation

In this talk, we introduce a computer-assisted technique for the analysis of periodic orbits of ill-posed PDEs. As a case study, our proposed method is applied to the Boussinesq equation, which has been investigated extensively because of its role in the theory of shallow water waves. The idea is to use the symmetry of the solutions and a Newton-Kantorovich type argument (the radii polynomial approach), to obtain rigorous proofs of existence of the periodic orbits in a weighted ell-one Banach space of space-time Fourier coefficients with geometric decay. We present computer-assisted proofs of existence of periodic orbits at different parameter values. This is joint work with Marcio Gameiro (USP, Brazil) and Roberto Castelli (VU Amsterdam, Netherlands).

KONSTANTIN MISCHAIKOW, Rutgers University
A combinatorial/algebraic topological approach to dynamics of regulatory networks.

Models of multiscale systems, such as those encountered in systems biology, are often characterized by heuristic nonlinearities and poorly defined parameters. Furthermore, it is typically not possible to obtain precise experimental data for these systems. Nevertheless, verification of the models requires the ability to obtain meaningful dynamical structures that can be compared quantitatively with the experimental data. With this in mind we present an approach to modeling dynamics that is based on a purely topological approach to dynamics. We will describe these ideas in the context of models for gene regulatory networks.

MAXIME MURRAY, Université Laval
Travelling waves for the suspended bridge equation

In this talk, we present a computer-assisted technique to prove the existence of travelling waves for the suspended bridge equation for a continuous range of parameter values. The idea is to express the travelling waves as solutions of a boundary value problem (BVP) with the boundary values in the stable and unstable manifolds. The technique uses the parametrization method for invariant manifolds and Chebyshev series, and the BVP is solved in a Banach space of exponentially decaying Chebyshev coefficients. The proof relies on using the uniform contraction principle, with the help of the radii polynomial approach. We will discuss the advantages and the difficulties of our proposed approach. This is joint work with J.B. van den Berg (VU Amsterdam, Netherlands), M. Breden (ENS Cachan, France and Université Laval, Canada) and J.-P. Lessard (Université Laval, Canada).

VIDIT NANDA, University of Pennsylvania
Homotopy-inference for functions

We survey the work of Niyogi, Smale and Weinberger which provides explicit bounds on size of a uniformly random point-sample required to reconstruct the homotopy type of an underlying compact Riemannian manifold with high confidence. We also describe an analogous result for Lipschitz-continuous functions between such manifolds: one can recover the action on homotopy of such a function using only finitely many evaluations. This is joint work with Steve Ferry and Konstantin Mischaikow.

EVELYN SANDER, George Mason University
Chaos and quasiperiodicity

Periodicity, quasiperiodicity, and chaos are the types of typically observed in general dynamical systems. The Birkhoff Ergodic Theorem asserts that the Birkhoff time average, $\Sigma_{n=0}^{N-1} f(x_n)/N$ of a function f along a length N ergodic trajectory (x_n) of a function T converges to the space average $\int f d\mu$, where μ is the unique invariant probability measure for T. This relationship between the time and space averages is powerful, since often a time series is the only information available. However, the convergence of the Birkhoff average is slow, with an error of order N^{-1} for a length N trajectory. We present a modified Birkhoff average technique by giving very small weights to the terms to $f(x_n)$ when n is near 0 or $N-1$. Our method is to calculate $\Sigma_{n=0}^{N-1} w(n/N) f(x_n)$, where the weighting function w vanishes smoothly at the ends 0 and 1. This method is a significant improvement: when (x_n) is a trajectory on a quasiperiodic torus and f and T are infinitely-many times differentiable, our method of weighted Birkhoff average converges exponentially fast to $\int f d\mu$ with respect to the number of iterates N, *i.e.* with error decaying faster than N^{-m} for every integer m. As a result of this speed, we are able to obtain high precision values for $\int f d\mu$ with relatively low computational cost. Our weighted Birkhoff average is a powerful computational tool for computing rotation numbers and conjugacies. This is joint work with Suddhasattwa Das, Yoshitaka Saiki and James Yorke.

JAN BOUWE VAN DEN BERG, VU Amsterdam
Rigorous numerics for some pattern formation problems

This talk is about applications of recently developed techniques from rigorous computational dynamics to pattern formation phenomena. We discuss the differences and similarities in the analytic setup of three examples, namely radially symmetric spots in the Swift-Hohenberg model, transitions between hexagonal spots and stripe patterns, and phase separation in diblock copolymers. These examples, which entail both ODEs and PDEs, also showcase the interplay between rigorous numerical methods and asymptotic techniques.

LENNAERT VAN VEEN, University of Ontario Institute of Technology
Equilibria and periodic orbits in 3D Navier-Stokes flow on a periodic domain

In this collaboration with Susumu Goto of Osaka University, we compute simple invariant solutions in incompressible Navier-Stokes flow in a periodic box. We use several different external body forces to input energy. Depending on the forcing, the transition from laminar to turbulent flow can be sub or super critical. While the invariant solutions are all approximate, I will focus in particular on equilibria and periodic orbits at relatively low Reynolds numbers, which may be amenable to rigorous computation and continuation.

THOMAS WANNER, George Mason University
Rigorous Validation of Isolating Blocks for Flows

In this talk, we present a new method for finding and rigorously verifying a special type of index pairs for finite-dimensional flows, namely isolating blocks and their exit sets. Our method makes use of a recently developed adaptive algorithm for rigorously determining the location of nodal sets of smooth functions, which combines an adaptive subdivision technique with interval arithmetic. By characterizing an exit set as a nodal domain, we are able to determine a valid index pair and proceed to compute its Conley index. Our method is illustrated using several examples for three-dimensional flows.

JF WILLIAMS, Simon Fraser University
Rigorous numerics for BVPs on infinite domains

This talk will present some examples of solving Boundary Value Problems on infinite domains using rigorous numerics. The idea is to use a Newton-Kantorovich type argument on a transformed problem. We will discuss several choices for transforming the problem to a finite domain and explore the relationship between the numerical and functional analytic frameworks required to perform rigorous numerics.

MICHAEL YAMPOLSKY, University of Toronto
Geometrization of branched coverings of the sphere and decidability of Thurston equivalence

I will discuss a recent joint work with N. Selinger on constructive geometrization of branched coverings of the 2-sphere. I will further describe the connection between geometrization and the general question of algorithmic decidability of Thurston equivalence, and will present a new decidability result obtained jointly with Selinger, which generalizes my previous work with M. Braverman and S. Bonnot.