KARL DILCHER, Dalhousie University

Generalized Fermat numbers and congruences for Gauss factorials

We define a Gauss factorial $N_n!$ to be the product of all positive integers up to N that are relatively prime to $n \in \mathbb{N}$. We consider the Gauss factorials $\lfloor \frac{n-1}{M} \rfloor_n!$ for $M = 3$ and 6, where the case of n having exactly one prime factor of the form $p \equiv 1 \pmod 6$ is of particular interest. A fundamental role is played by primes with the property that the order of $\frac{p-1}{3}!$ modulo p is a power of 2 or 3 times a power of 2; we call them Jacobi primes. Our main results are characterizations of those $n \equiv \pm 1 \pmod M$ of the above form that satisfy $\lfloor \frac{n-1}{M} \rfloor_n! \equiv 1 \pmod n$, $M = 3$ or 6, in terms of Jacobi primes and certain prime factors of generalized Fermat numbers. (Joint work with John Cosgrave).