VINCENT PILAUD, CNRS & LIX, École Polytechnique
Brick polytopes, lattice quotients, and Hopf algebras

This talk is motivated by the deep connections between the combinatorial properties of permutations, binary trees, and binary sequences. Namely, classical surjections from permutations to binary trees (BST insertion) and from binary trees to binary sequences (canopy) yield:

- lattice morphisms from the weak order, via the Tamari lattice, to the boolean lattice;
- normal fan coarsenings from the permutahedron, via Loday's associahedron, to the parallelepiped generated by the simple roots $\mathbf{e}_{i+1} - \mathbf{e}_i$;
- Hopf algebra inclusions from Malvenuto-Reutenauer's algebra, via Loday-Ronco's algebra, to Solomon's descent algebra.

In this talk, we present an extension of this framework to acyclic k-triangulations of a convex (n+2k)-gon, or equivalently to acyclic pipe dreams for the permutation $(1, \ldots, k, n+k, \ldots, k+1, n+k+1, \ldots, n+2k)$. These objects are in bijection with the classes of the congruence of the weak order on $\mathfrak{S}_n$ defined as the transitive closure of the rewriting rule $UacV_1b_1 \cdots V_kb_kW \equiv^k UcaV_1b_1 \cdots V_kb_kW$, for letters $a < b_1, \ldots, b_k < c$ and words $U, V_1, \ldots, V_k, W$ on $[n]$. It enables us to transport the known lattice and Hopf algebra structures from the congruence classes of $\equiv^k$ to these acyclic pipe dreams. We will describe the cover relations in this lattice and the product and coproduct of this algebra in terms of pipe dreams. We will also recall the connection to the geometry of the brick polytope.