Toric and Combinatorial Algebraic Geometry
Géométrie algébrique torique et combinatoire
(Org: **Eric Katz** (Waterloo) and/et **Jessie Yang** (McMaster))

DAVE ANDERSON, The Ohio State University
Beyond the determinantal formula of Schubert calculus

In 1974, Kempf and Laksov proved a generalization of Giambelli's formula: they expressed the cohomology class of a degeneracy locus as a determinant in Chern classes of the vector bundles involved. In modern terms, this is a formula for the *equivariant* class of a Schubert variety in the Grassmannian. I will discuss some recent results that explain why the occurrence of such determinants is natural. This perspective leads to extensions to other homogeneous spaces, as well as to applications to quantum cohomology.

DUSTIN CARTWRIGHT, University of Tennessee
Tree compactifications of the moduli space of genus 0 curves

The moduli space of genus zero curves with marked points has a well-known compactification, due to Deligne and Mumford. This compactification can be constructed as a closure of the moduli space of smooth curves inside a toric variety. I will talk about an broader class of compactifications which can be formed by taking closures inside other toric varieties. These compactifications are indexed by certain compatible families of trees.

GRAHAM DENHAM, University of Western Ontario
Coordinate rings of compactified arrangements

The intersection of a linear space with a complex torus $(\mathbb{C}^*)^n$ is the complement of a complex hyperplane arrangement. Closures of these in toric varieties give a range of interesting compactifications, including $\overline{M}_{0,n}$.

I will report on work in progress with Greg Smith (Queen's) and Uli Walther (Purdue) that examines the coordinate rings of such compactifications in their most natural projective embeddings.

KAEL DIXON, McGill University
How black holes give counter-examples to the convexity of the moment map.

A symplectic manifold (M^{2n}, ω) is toric if it admits a Hamiltonian action of the real torus $(S^1)^n$. It then admits a moment map, which identifies the orbit space of the action as a subset of the dual Lie algebra of the torus. It is a standard result that if M is complete, then the moment map is convex.

In this talk, I will show how the Riemannian analogues of the Kerr family of spacetimes, corresponding to rotating black holes, can be given toric structures. I will explain how the moment map loses convexity in the region corresponding to the ring singularity in the interior of the black hole.

CHARLES DORAN, University of Alberta
K3 fibrations on Calabi-Yau threefolds and Landau-Ginzburg models of Fano threefolds

Calabi-Yau threefolds can be constructed as smoothings of unions of pairs of blown-up Fano threefolds. It is well known that this construction should be related to K3 surface fibrations on the mirror Calabi-Yau threefold. I will describe in several cases how mirror Calabi-Yau threefolds can be constructed from the Landau-Ginzburg models of the Fano threefolds with which we began.

MATTHIAS FRANZ, University of Western Ontario
Syzygies in equivariant cohomology for non-abelian Lie groups

We extend the work of Allday-Franz-Puppe on syzygies in equivariant cohomology from tori to arbitrary compact connected Lie groups G. In particular, we show that for a compact orientable G-manifold X the analogue of the Chang-Skjelbred sequence is exact if and only if the equivariant cohomology of X is reflexive, if and only if the equivariant Poincaré pairing for X is perfect. A crucial step of the proof is to show that the equivariant cohomology modules arising from the orbit filtration of X are Cohen-Macaulay. We also discuss how syzygies behave with respect to restriction of the action to and induction from a maximal torus. Big polygon spaces lead again to important examples.

LISA JEFFREY, University of Toronto
A Hamiltonian circle action on the triple reduced product of coadjoint orbits of SU(3)

The fundamental group of the three-punctured sphere is the free group on two generators – or, more symmetrically, the group on three generators with one relation (so that the product of the generators equals the identity). Representations of this group in compact Lie groups have been much studied (as a building block in the theory of flat connections on 2-manifolds). This is related to the Verlinde algebra.

Analogously one may study the symplectic quotient at 0 of the product of three coadjoint orbits of a Lie group (the triple reduced product). (This is the same as the space of representations of the fundamental group of the three-punctured sphere when the three orbits are very close to zero, close enough that the exponential map gives a bijection.) For regular orbits of $G = SU(3)$ we have constructed this symplectic quotient explicitly and confirmed that is it a 2-sphere, as one would expect on general grounds. We describe how to explicitly find a function whose Hamiltonian flow gives an S^1 action on this 2-sphere.

(Joint work in progress with Steven Rayan, Gouri Seal, Paul Selick and Jonathan Weitsman)

CAROLINE JUNKINS, Western University
The Steinberg basis and the twisted gamma-filtration

For a simple linear algebraic group G and a projective G-homogeneous variety X, a result of Panin relates the indices of the Tits algebras of G to non-trivial torsion elements in the γ-filtration on the Grothendieck group $K_0(X)$. Such torsion elements are of interest due to their close connection to the Chow group of X and to the set of cohomological invariants of G. The Steinberg basis of $K_0(X)$ provides an explicit set of generators for the γ-filtration, however the relations are not easily computed.

A tool introduced by Zainoulline called the twisted γ-filtration acts as a surjective image of the γ-filtration, with explicit sets of both generators and relations. In this talk we use the twisted γ-filtration to construct torsion elements in the degree 2 component of the γ-filtration for groups of inner type D_{2n}.

YAEL KARSHON, University of Toronto
Non-compact symplectic toric manifolds

I will report on joint work with Eugene Lerman in which we extend Delzant's classification of compact symplectic toric manifolds to the non-compact case. The quotient of a symplectic toric manifold by the torus action is a manifold with corners, Q. The classification is in terms of a "unimodular local embedding" from Q to the dual of the Lie algebra of the torus, plus a degree two cohomology class on Q. The main technical issue is to construct the smooth structure "upstairs" when Q has infinitely many facets.

KALLE KARU, The University of British Columbia
Non-finitely generated Cox rings

Cox rings generalize the homogeneous coordinate ring of a toric variety. The main problem in the theory of Cox rings is to determine if they are finitely generated.

Goto, Nishida and Watanabe (1994) gave an infinite sequence of weighted projective planes blown up at a point, so that the resulting varieties do not have finitely generated Cox rings. Castravet and Tevelev (2013) used these varieties to prove that the Cox ring of the moduli space of rational stable n-pointed curves $\overline{M}_{0,n}$ is not finitely generated if $n \geq 134$.

We extend the family of weighted projective planes given by Goto, Nishida and Watanabe to more examples of non-finitely generated Cox rings. Then using the reduction method of Castavet and Tevelev, we prove that the Cox ring of $\overline{M}_{0,n}$ is not finitely generated if $n \geq 13$.

This is a joint work with Jose Gonzalez.

KIUMARS KAVEH, University of Pittsburgh
Toric degenerations and applications

We discuss general Rees algebra constructions from commutative algebra deforming an algebra to the graded of a filtration. In specific we talk about such deformations associated to a valuation on the homogeneous coordinate ring of a variety (considered by Teissier). This gives a "toric degeneration" of the variety. We will then explain how one can use these toric degenerations in the context of symplectic/Kahler geometry, in particular to construct integrable systems/Hamiltonian torus actions on smooth projective varieties, as well as to construct convergence of real polarization to complex polarization in geometric quantization.

ASKOLD KHOVANSKII, University of Toronto
Newton polyhedra and irreducible components of complete intersection

Consider a variety X defined in $(\mathbb{C}^*)^n$ by a generic system of equations with given Newton polyhedra. It is known that many "natural" discrete invariants of X can be explicitly computed in terms of Newton polyhedra. I will talk about the number $b_0(X)$ of irreducible components of X. There are two classical results about $b_0(X)$. First, if $\dim X = 0$ then by Bernstein-Kouchnirenko theorem $b_0(X)$ is equal to the mixed volume of Newton polyhedra multiplied by $n!$. Second, if $\dim X > 0$ and all Newton polyhedra have the biggest possible dimension n then $b_0(X) = 1$. I will explain how to compute $b_0(X)$ in general case. One extra result. It turns out that each component of X can be defined by a generic system of equations whose Newton polyhedra can be constructed explicitly. So a natural discrete invariant of each component can be computed explicitly (such invariant takes the same value at all components of X).

JOEL LEMAY, University of Ottawa
Geometric Fock Space Representations of Affine sl(n) and gl(n)

Fock space has a basis naturally enumerated by Young diagrams. There exists a well-known combinatorial representation of affine sl(n) on Fock space, where the Chevalley generators act by adding and removing boxes. This can be extended to an action of affine gl(n), resulting in a combinatorial realization of the so-called basic representation. In this talk, we describe a geometric realization of Fock space using the equivariant cohomology of Hilbert schemes and Nakajima quiver varieties. In particular, we show how to geometrically describe the action of affine sl(n) and gl(n) using "geometric operators" arising from the top nonvanishing Chern classes of certain equivariant vector bundles. Our description, which is more general than those previously appearing in the literature, yields a geometric realization of all the vertex operator realizations of the basic representation.

CHRISTOPHER MANON, George Mason University.
Toric geometry of moduli spaces of principal bundles on a curve.

For C a smooth projective curve, and G a simple, simply connected complex group, let $M_C(G)$ be the moduli space of semistable $G-$principal bundles on C. As the curve C moves in the moduli $\mathcal{M}_g$ of smooth curves, the spaces $M_C(G)$ are known to define a flat family of schemes, and this family can be extended to the Deligne-Mumford compactification $\bar{\mathcal{M}}_g$. We

describe the geometry of the fibers of this family which appear at the stable boundary, in particular we discuss a recent result which shows that the fibers over maximally singular curves contain an important and ubiquitous moduli space, the free group character variety $\mathcal{X}(F_g, G)$, as a dense, open subspace. The latter is a moduli space of representations of the free group F_g in G, and naturally appears as an object of interest in Teichmüller theory, the theory of geometric structures, and the theory of Higgs bundles. For $G = SL_2(\mathbb{C})$ and $SL_3(\mathbb{C})$ we describe maximal rank valuations on the coordinate rings of these spaces, and how the associated Newton-Okounkov polyhedra can be used to study the geometry of both $\mathcal{X}(F_g, G)$ and $M_C(G)$.

FARBOD SHOKRIEH, Cornell University
Faithful Tropicalization of Abelian Varieties

We study faithful tropicalization of abelian varieties. For an abelian variety, the skeleton (in the sense of Berkovich) is a real torus with an "integral structure. For totally degenerate Jacobians, we give an explicit faithful tropicalization in terms of non-Archimedean and tropical theta functions. The solution relies on interesting combinatorial facts about lattices and Voronoi decompositions. (Ongoing joint work with Tyler Foster (University of Michigan), Joe Rabinoff (Georgia Tech), and Alejandro Soto (KU Leuven))

HUGH THOMAS, University of New Brunswick
Monodromy for the quintic mirror

The mirror to the quintic in $\mathbb{P}^4$ is the Dwork family X_p of Calabi-Yau 3-folds over a thrice-punctured sphere. As p moves in a loop around each of the three punctures, we can parallel transport classes in $H^3(X_p, \mathbb{Q})$, and observe the monodromy. $H^3(X_p, \mathbb{Q})$ is four-dimensional, and the monodromy can be expressed by matrices in $\mathrm{Sp}(4, \mathbb{Z})$. We showed that these matrices generate a subgroup isomorphic to the free product $\mathbb{Z}/5 * \mathbb{Z}$. The subgroup is dense in $\mathrm{Sp}(4, \mathbb{Z})$, but it was not known whether or not it is of finite index. The fact that it is a free product implies that it cannot be. The monodromy is thus "thin"; this is the first example known of thin monodromy arising algebro-geometrically inside a Lie group of real rank greater than 1.

The Dwork family is one of 14 similar families of CY 3-folds; our methods establish similar results for 7 of the 14 families. For the other 7, it has recently been shown by Singh-Venkataramana and Singh that the monodromy is of finite index, so our result is best possible. This talk is based on joint work with Chris Brav, "Thin monodromy in $\mathrm{Sp}(4, \mathbb{Z})$," Compositio Mathematica, 2014.