Stochastic Models and Applications
Modèles stochastiques et applications
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LOUIGI ADDARIO-BERRY, McGill University
Random maps and their cores

Let Q be a large random quadrangulation let R be its largest simple subgraph and P be its second-largest simple subgraph. Then $|R|/|Q|$ is concentrated near a fixed integer $\alpha \in (0, 1)$, and $|P|/|Q|$ is very likely close to zero; in other words, large quadrangulations with high probability have a unique simple "core" of linear size, decorated with small (sub-linear size) attachments. We use this picture to show that the pair (Q, R), after suitable rescaling, converges in the Gromov-Hausdorff-Prokhorov sense to a limit (M, M), where M is a random variable with the law of the Brownian map. This requires showing that the distribution of mass in Q and R is asymptotically equal, which we establish through an "invariance principle for exchangeable, asymptotically negligible attachments" for measured metric spaces.

LOUIS-PIERRE ARGUIN, Université de Montréal
Probabilistic approach for the maxima of the Riemann Zeta function on the critical line

A recent conjecture of Fyodorov, Hiary & Keating states that the maxima of the Riemann Zeta function on a bounded interval of the critical line behave similarly to the maxima of a specific class of Gaussian fields, the so-called log-correlated Gaussian fields. These include important examples such as branching Brownian motion and the 2D Gaussian free field. In this talk, we will highlight the connections between the number theory problem and the probabilistic models. We will outline the proof of the conjecture in the case of a randomized model of the Zeta function. We will discuss possible approaches to the problem for the function itself. This is joint work with D. Belius (NYU) and A. Harper (Cambridge).

RALUCA BALAN, University of Ottawa
Intermittency for the stochastic wave and heat equations with fractional noise in time

Stochastic partial differential equations (SPDEs) are mathematical objects that are used for modeling the behaviour of physical phenomena which evolve simultaneously in space and time, and are subject to random perturbations. A key component of an SPDE which determines the properties of the solution is the underlying noise process. An important problem is to study the impact of the noise on the behavior of the solution. In the study of SPDEs using the random field approach, the noise is typically given by a generalization of the Brownian motion, called the space-time white noise. In this talk, we consider the stochastic heat and wave equations driven by a Gaussian noise which is homogeneous in space and behaves in time like a fractional Brownian motion with index $H > 1/2$. We study a property of the solution $u(t, x)$ called intermittency. This property was introduced by physicists as a measure for describing the asymptotic behaviour of the moments of $u(t, x)$ as $t \to \infty$. Roughly speaking, u is "weakly intermittent" if the moments of $u(t, x)$ grow as $\exp(ct)$ for some $c > 0$. It is known that the solution of the heat (or wave) equation driven by space-time white noise is weakly intermittent. We show that when the noise is fractional in time and homogeneous in space, the solution u is "weakly ρ-intermittent", in the sense that the moments of $u(t, x)$ grow as $\exp(ct^{\rho})$, where $\rho > 0$ depends on the parameters of the noise.
This talk is based on joint work with Daniel Conus (Lehigh University).

PHELIM BOYLE, Wilfrid Laurier University
Beyond Perron Frobenius

The classical Perron-Frobenius theorem provides a sufficient condition for the dominant eigenvector of an n by n matrix to be positive. The condition is that all the matrix elements are positive. An extension of this result has a direct application in finance. The dominant eigenvector of the correlation matrix of stock returns can proxy the market portfolio. As the market portfolio must have positive weights we are interested in the conditions under which elements of this eigenvector are positive. It turns out that one can have some negative elements in the correlation matrix and the matrix can still have a positive dominant eigenvector. We analyze these conditions and this leads to extensions of the Perron Frobenius theorem

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DONALD A. DAWSON, Carleton University
Random walk, percolation and branching systems on the hierarchical group

Spatial population models have been intensively studied for many years. Classical branching systems are well understood in homogeneous spaces such as $\mathbb{R}^2$ or $\mathbb{Z}^d$. Much less is known about more complex systems such as catalytic branching systems, in particular mutually catalytic systems, even in homogeneous spaces and much less is known in random media. The purpose of this lecture is to explain some recent work in this direction and some conjectures and open problems. As a starting point we introduce the hierarchical group giving some motivation and comparison to the Euclidean group. We then consider branching systems in which the spatial movement is given by a random walk in these spaces and the role of the potential theoretic properties, in particular the degree of transience-recurrence of the random walk. We then consider the question of percolation for a related class of random graphs embedded in these spaces and end with an open problem concerning the properties of branching systems on these percolation clusters serving as a random medium. This talk is based on joint projects with Luis Gorostiza and Andreas Greven.

STEFANO FAVARO, University of Torino
A new tool for nonparametric estimation of species variety with Gibbs-type priors

Bayesian nonparametric inference for species sampling problems concerns with the estimation, conditional on an initial observed sample, of the species variety featured by an additional unobserved sample. Within the framework of Gibbs-type priors, we introduce a new tool for estimating species variety when the additional sample is required to be very large and the implementation of exact Bayesian nonparametric procedures is prevented by cumbersome computation. Our result is illustrated through a simulation study and the analysis of a real dataset in linguistics.

RAFAL KULIK, University of Ottawa
Heavy tailed time series with extremal independence

We consider heavy tailed time series whose finite-dimensional distributions are extremally independent in the sense that extremely large values cannot be observed consecutively. This calls for methods beyond the classical multivariate extreme value theory which is convenient only for extremally dependent multivariate distributions. We use the Conditional Extreme Value approach to study the effect of an extreme value at time zero on the future of the time series. In formal terms, we study the limiting conditional distribution of future observations given an extreme value at time zero. To this purpose, we introduce conditional scaling functions and conditional scaling exponents. We compute these quantities for a variety of models, including Markov chains, exponential autoregressive models, stochastic volatility models with heavy tailed innovations or volatilities.

DELI LI, Lakehead University
A Characterization of a New Type of Strong Law of Large Numbers

Let $0 < p < 2$ and $1 \leq q < \infty$. Let $\{X_n;\ n \geq 1\}$ be a sequence of independent copies of a real-valued random variable X and set $S_n = X_1 + \cdots + X_n,\ n \geq 1$. We say X satisfies the (p,q)-*type strong law of large numbers* (and write $X \in SLLN(p,q)$) if $\sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{|S_n|}{n^{1/p}} \right)^q < \infty$ almost surely. This talk is devoted to a characterization of $X \in SLLN(p,q)$. By applying results obtained from the new versions of the classical Lévy, Ottaviani, and Hoffmann-Jørgensen (1974) inequalities proved by Li and Rosalsky (2013) and by using techniques developed by Hechner (2009) and Hechner and Heinkel (2010), we obtain sets of necessary and sufficient conditions for $X \in SLLN(p,q)$ for the six cases: $1 \leq q < p < 2$, $1 < p = q < 2$, $1 < p < 2$ and $q > p$, $q = p = 1$, $p = 1 < q$, and $0 < p < 1 \leq q$. The necessary and sufficient conditions for $X \in SLLN(p,1)$ have been discovered by Li, Qi, and Rosalsky (2011). Versions of above results in a Banach space setting are also given. Illustrative examples are presented.

NEAL MADRAS, York University
Random 312-Avoiding Permutations

A *pattern* of length k is simply a permutation of $\{1, .., k\}$. A permutation of $\{1, ..., N\}$ (for $N > k$) is said to avoid a specific pattern P if the (long) permutation has no subsequence of k elements that appears in the same relative order as P. (E.g. the permutation (2463175) does not avoid the pattern (312) because the permutation contains the subsequence (615).) Pattern avoidance has been extensively studied by combinatorialists.

Simulations suggest intriguing structural properties of permutations generated uniformly at random from $S_N[312]$, the subset of permutations of $\{1, .., N\}$ that avoid 312. To elucidate these properties, we obtain exact and asymptotic probabilities that the i^{th} entry of such a permutation equals j, as well as joint probabilities of such events. We also find that for large N, a cluster of points "below the diagonal" in a graph of such a permutation looks like the trajectory of a directed random walk with infinite mean.

This is joint work with Lerna Pehlivan.

DON L. MCLEISH, University of Waterloo
Convergence of the Discrete Variance Swap in Time-Homogeneous Diffusion Models

Discretely sampled variance swaps are financial instruments whose price depends on the observed volatility or variance of an underlying. They are traded in the market, and usually the fair strikes of continuously sampled variance swaps are used to approximate their discrete counterparts. There has been work (Jarrow, Kchia, Larsson and Protter (2013)) discussing conditions under which this approximation is valid for semi-martingales, and also several papers proposing studying explicit formulae of discretely sampled variance swaps for specific stochastic volatility models, such as the Heston stochastic volatility model (Broadie and Jain (2008)), the Hull-White and the Schobel-Zhu stochastic volatility models (Bernard and Cui (2014)). For stochastic volatility models based on time-homogeneous diffusions, we provide a simple necessary and sufficient condition for the discretely sampled fair strike of a variance swap to converge to the continuously sampled fair strike, extending Theorem 3:8 of Jarrow, Kchia, Larsson and Protter (2013). We also give conditions (not based on asymptotics) when the fair strike of the discrete variance swap is higher than the continuous one and discuss the convex order conjecture proposed by Griessler and Keller-Ressel (2014) in this context. This is joint work with Carole Bernard, University of Waterloo, and Zhenyu Cui, Brooklyn College of the City University of New York.

CLARENCE SIMARD, UQAM
General model for limit order book and market orders

We introduce a general model for the structure and the dynamic of the limit order book in continuous time which includes the properties of depth, tightness and resilience. Our starting point is using random processes with value in the space of continuous functions to model the cost of transactions instead of modeling the behaviour of the asset price. The portfolio value takes into account the opposing forces between market orders, which deplete the limit order book, and the arrival of new limit orders. We prove that the existence of some equivalent probability measure is sufficient to rule out arbitrage and that the converse cannot hold in general. This result generalizes similar non-arbitrage theorems found in the literature on limit order book as well as the sufficiency part of the first fundamental of asset pricing.

WEI SUN, Concordia University
New criteria for Hunt's hypothesis (H) of Levy processes

A Markov process X is said to satisfy Hunt's hypothesis (H) if every semi-polar set is polar. Roughly speaking, this means that if a set A cannot be immediately hit by X for any starting point, then A will never be hit by X. About fifty years ago, Professor

R.K. Getoor conjectured that essentially all Levy processes satisfy (H). In this talk, we present novel necessary and sufficient conditions for the validity of (H) of Levy processes. As applications, we obtain new examples of Levy processes satisfying (H). Moreover, we show that a general class of pure jump subordinators can be decomposed into the summation of two independent subordinators satisfying (H).

XIAOWEN ZHOU, Concordia University
Some Support Properties of Λ-Fleming-Viot Processes with Brownian Spatial Motion

A Fleming-Viot process is a probability-measure-valued stochastic process for mathematical population genetics. It describes the evolution of relative frequencies for different types of alleles in a large population that undergoes reproduction and mutation.

In this talk I first briefly review the Λ-coalescent of multiple collisions and the lookdown representation of Donnelly and Kurtz for Λ-Fleming-Viot process with Brownian spatial motion. I then present several support properties obtained in [1,2,3] on the Λ-Fleming-Viot random measure. These properties include the compact support property, the modulus of continuity, Hausdorff dimensions and the disconnectedness. The lookdown representation is crucial in showing all these results. If time allows I will also introduce some recent work in progress.

References

[1] H. Liu and X. Zhou (2012). Compact support property of the Λ-Fleming-Viot process with underlying Brownian motion. *Electronic Journal of Probability*, *17*, No. 73, 1-20.

[2] H. Liu and X. Zhou (2013). Some support properties for a class of Λ-Fleming-Viot processes. To appear. *Annales de L'Institut Henri Poincaré (B) Probabilités et Statistiques*. Available at http://arxiv.org/abs/1307.3990.

[3] X. Zhou (2014). On criteria of disconnectedness for Λ-Fleming-Viot support. *Electronic Communications in Probability*, *19*, No 53, 1-16.

YOUZHOU ZHOU, Zhongnan University of Economics and Law
Some Large Deviation Principles and Law of Large Numbers for Random Energy Model

Random Energy Model (in short REM) is a toy model for spin glasses, a special state for magnetic materials below a critical temperature T_c. The Poisson-Dirichlet distribution $P(\alpha, 0)$, where $\alpha = \frac{T}{T_c}$, indicates the probability weights of infinitely many pure states in REM. In this talk, large deviations for $P(\alpha, 0)$ as $T \to T_c(i.e.\alpha \to 1)$ is considered. Moreover, we will also consider large deviations for

$$\pi_{\alpha,\lambda}(dp) = C_{\alpha,\lambda} \exp\left\{\lambda(\alpha)\sum_{i=1}^{\infty} p_i^2\right\} PD(\alpha, 0)(dp),$$

where $C_{\alpha,\lambda}$ is a normalizing constant and $\alpha \to 1$. Here $\pi_{\alpha,\lambda}$ resembles the Poisson-Dirichlet distribution with selection in population genetics. Interestingly the large deviations for $\pi_{\alpha,\lambda}$ reveals phase transition. The weak law of large numbers in critical case is also covered in this talk.