Geometric Discretization Methods and Adaptivity
Méthodes de discrétisation géométrique et adaptabilité

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HARBIR ANTIL, George Mason University

Optimal control of a free boundary problem

We will consider a PDE constrained optimization problem governed by a free boundary problem. The state system is based on coupling the Laplace equation in the bulk with a Young-Laplace equation on the free boundary to account for surface tension. This amounts to solving a second order system both in the bulk and on the interface. Our analysis hinges on a convex constraint on the control such that the state constraints are always satisfied. Using only first order regularity we show that the control to state operator is twice Fréchet differentiable. We improve slightly the regularity of the state variables and exploit this to show existence of a control together with second order sufficient optimality conditions. We prove that the state and adjoint system have the requisite regularity for the error analysis (strong solutions). We discretize the state, adjoint and control variables via piecewise linear finite elements and show optimal $O(h)$ error estimates for all variables, including the control.

GERARD AWANOU, University of Illinois at Chicago

Analysis of numerical methods for the Monge-Ampère equation

The Monge-Ampère equation is a nonlinear partial differential equation with a geometric theory due to Aleksandrov. It appears in a wide range of applications, e.g. optimal transportation and reflector design. Solutions of the Monge-Ampère equation are in general not smooth, and hence difficult to compute with standard discretizations. I will review a large class of methods proposed so far, from the point of view of structure preserving discretizations. I will discuss how this new point of view leads to an analysis of the theoretical convergence properties of the methods.

ANIL HIRANI, University of Illinois at Urbana-Champaign

New Spaces for DEC

Discrete Exterior Calculus (DEC) is a combinatorial discretization of exterior calculus. I will first show how the primal-dual discretization of DEC can be viewed in terms of the finite element exterior calculus framework. Then I will describe a family of new piecewise constant differential forms spaces for interpreting the mass and stiffness matrices of DEC. This is joint work with Alan Demlow (Texas A&M Math) and Kaushik Kalyanaraman (UIUC CS).

NICHOLAS KEVLAHAN, McMaster University

A dynamically adaptive wavelet-based method for the shallow water equations on the icosahedral sphere

This talk presents a dynamically adaptive wavelet method for the shallow water equations on the staggered hexagonal C-grid on the sphere. The adaptive grid hierarchy is a dyadic subdivision of the icosahedron, which is optimized to ensure good geometric properties. Distinct biorthogonal second generation wavelet transforms are developed for the pressure and the velocity, together with compatible restriction operators to ensures discrete mass conservation and no numerical generation of vorticity. Coastlines are introduced by a new volume penalization method of the shallow water equations which ensure inertia-gravity waves are reflected physically, and that no-slip boundary conditions are imposed for the horizontal velocity. The code is fully parallelized using mpi, and we demonstrate good weak parallel scaling to at least 1000 processors. The efficiency and accuracy of the method are verified by applying it to a tsunami-type inertia-gravity wave with full topography, to wind-driven gyre flow and to homogeneous rotating turbulence. Even in the unfavourable case of homogeneous turbulence significant savings in the number of degrees of freedom are achieved by the adaptivity.

SCOTT MACLACHLAN, Memorial University of Newfoundland
First-order system Petrov-Galerkin discretization for a singularly perturbed reaction-diffusion problem

This talk presents a Petrov-Galerkin finite-element discretization of a singularly perturbed reaction-diffusion equation posed on the unit square. We extend the work of Lin and Stynes (2012), who suggest that the natural energy norm (associated with a standard Galerkin approach) is not an appropriate setting for analyzing such problems. They propose a method for which the natural norm is "balanced", reflecting important features of the continuum solution, but which requires discretization in an H(div) space. Here, in the style of a first-order system least squares (FOSLS) method, we extend their approach by introducing an additional constraint that simplifies the associated finite-element space and the resulting analysis. We prove robust convergence in a balanced norm on a mesh with a priori adaptation, presenting supporting numerical results and demonstrating optimal solution of the resulting linear systems using multigrid methods.

ANDY WAN, McGill University
Adaptive space-time finite element method for the p-curl problem from high temperature superconductivity

In applications of high temperature superconductors, a nonlinear eddy current problem, known as the p-curl problem, is often used to model electromagnetic properties of superconducting materials. Due to similarities with the parabolic p-Laplacian, solutions of this problem can exhibit sharp gradients which move in time. Moreover, the development of superconducting devices has been hindered by the computational difficulties at resolving these moving fronts. In this talk, we discuss an adaptive space-time finite element method which can intrinsically reduce the number of degrees of freedom for the p-curl problem. Based on a Helmholtz-Weyl decomposition for the space $W_0^p(\text{curl})$ and a residual type argument, we show the proposed a posteriori error estimators provide an upper bound for the error and as well as for the error in an energy dissipation quantity, known as AC losses. This is joint work with Marc Laforest and Frédéric Sirois at Polytechnique de Montréal.

ANDY WAN, McGill University
The multiplier method to construct conservative finite difference schemes for ordinary and partial differential equations

We present the multiplier method of constructing conservative finite difference schemes for ordinary and partial differential equations. Given a system of differential equations possessing conservation laws, our approach is based on discretizing conservation law multipliers and their associated density and flux functions. We show that the proposed discretization is consistent for any order of accuracy and that by construction, discrete densities can be exactly conserved. In particular, the multiplier method does not require the system to possess a Hamiltonian or variational structure. Examples, including dissipative problems, are given to illustrate the method. This is joint work with Alexander Bihlo at Memorial University and Jean-Christophe Nave at McGill University.

KRIS VAN DER ZEE, The University of Nottingham
Stable discretization of gradient-flow diffuse-interface models

In this talk I will consider nonlinear parabolic PDE systems with diffuse interfaces, which exhibit a gradient-flow structure, e.g., Cahn-Hilliard-type models. Owing to the gradient structure, the evolution is fully described by an energy functional and dissipation operator. Crucial for its discretization are so-called gradient-stable schemes, which preserve the gradient structure at the time-discrete level. I will present a recently proposed 2nd-order gradient-stable scheme based on convex splitting and artificial stabilization. Numerical examples will be given with application to a gradient-flow tumor-growth model.