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Huygens' clocks revisited

The first reported observation of synchronization of coupled oscillators was by Christiaan Huygens in 1665. He observed that, if two of his clocks were weakly coupled, after a short time they synchronized with opposite displacements and velocities, i.e. "anti-phase synchronization". Huygens was not able to explain his observation and it has been a topic of study to this day.

Our contributions to this study are as follows. First, we observed that Huygens' clocks were identical and symmetrically coupled; that is, they had a ($\mathbb{Z}_2$) permutation symmetry. Second, we observed that Huygens had reduced the linear friction of each clock-oscillator to nearly zero; that is, each was close to a Hopf bifurcation. Then we carried out a general analysis of double Hopf bifurcation with Huygens symmetry, using equivariant normal forms. This study revealed a rich variety of dynamic behaviours, including both in-phase and anti-phase normal modes and pairs of mixed-mode phase-locked periodic solutions. A theorem based on topological degree theory establishes the existence of quasiperiodic solutions in an invariant 3-torus that resembles a 2-torus "toroidal breather". An Arnold tongue plays a fundamental role in the secondary bifurcations to either phase-locked periodic solutions or quasiperiodic solutions. Numerical analysis using Matlab extends the local bifurcation analysis to a more global picture. Finally, application of this general theory to Huygens' clocks predicts his observation of "anti-phase synchronization".

This is joint work with P. M. Kitanov and A. R. Willms.