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Comparing two versions of the reals using computability

I will speak on joint work with Gregory Igusa. There are two different structures to which we attach the name "reals": the ordered field of real numbers $\mathcal{R} = (\mathbb{R}, +, \cdot, 0, 1, <)$, and $\mathcal{W} = (P(\omega) \cup \omega, P(\omega), \omega, \in, S)$, where S is the successor function on ω. Noah Schweber defined a reducibility that lets us compare uncountable structures according to the difficulty of building a copy (where the copies need not be completed unless we pass to a generic extension of the set theoretic universe in which the structures become countable). According to Schweber's definition, $\mathcal{A} \leq^*_w \mathcal{B}$ if in a generic extension of V in which both $\mathcal{A}$ and $\mathcal{B}$ countable, every copy of $\mathcal{B}$ computes a copy of $\mathcal{A}$. It is not difficult to see that $\mathcal{W} \leq^*_w \mathcal{R}$. With some effort, we show that $\mathcal{R} \not\leq^*_w \mathcal{W}$. When the two structures become countable, they enumerate the same Scott set. There is a real closed field $\mathcal{R}^*$ such that $\mathcal{W} \equiv^*_w \mathcal{R}^*$. The extra computing power of $\mathcal{R}$ comes from the fact that it is Archimedean.