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Cohen 1-generics and the finite intersection principle

In 2012, Dzhafarov and Mummert discuss the proof-theoretic and computability-theoretic strength of a number of set-theoretic principles involving the axiom of choice. In particular, they discuss the content of the finite intersection principle: that given any set of sets, there is a subset that is maximal with respect to having the finite intersection property (every finite subset having nonempty intersection). We say that a Turing degree, $\mathbf{d}$, has FIP if given any computably presented set of computable reals $\{X_i : i \in \omega\}$, $\mathbf{d}$ can enumerate a set of indices S such that $\{X_i : i \in S\}$ is a realization of the finite intersection principle for the first set.

In recent work, Diamondstone, Downey, Greenberg and Turetsky prove that a degree is FIP if it computes a Cohen 1-generic, and that the converse holds in the Δ^0_2 case. We present a priority-free construction that directly ties 1-genericity to FIP, and that shows that the converse holds in general. This provides what might be the first instance of a classical theorem of mathematics whose computability theoretic strength aligns exactly with the ability to compute a 1-generic.

A more subtle priority argument also shows that the a priori weaker 2IP property is also equivalent to being able to compute a 1-generic, and hence to FIP.