MATS BOIJ, KTH - Royal Institute of Technology
Cones of Hilbert Functions

In a joint work with Gregory G. Smith we study the closed convex hull of various collections of Hilbert functions. In this work we focus on graded modules generated in degree zero over a standard graded polynomial ring. In this context, we completely describe the supporting hyperplanes and extreme rays for the cones generated by the Hilbert functions of all modules, all modules with bounded a-invariant, and all modules with bounded Castelnuovo-Mumford regularity. The first of these cones is infinite-dimensional and simplicial, the second is finite-dimensional but neither simplicial nor polyhedral, and the third is finite-dimensional and simplicial. We also give explicit linear bounds for the graded Betti numbers of modules with a given Hilbert function in the case of bounded Castelnuovo-Mumford regularity.