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On a class of Generalized Derivative Nonlinear Schrödinger Equations

We study a class of generalized Derivative Nonlinear Schrödinger (gDNLS) equations of the form $i\psi_t + \psi_{xx} + i|\psi|^{2\sigma}\psi_x = 0$ with $\sigma > 1$. When $\sigma = 1$, this equation reduces to the canonical DNLS equation that arises from magnetohydrodynamics as well as in studies of ultrashort optical pulses. The DNLS shares the same scaling properties (L^2 critical) as the quintic Nonlinear Schrödinger equation which is known to have finite time blow-up solutions. However, the long time existence/possible occurrence of singularities for the DNLS equation remains an open problem. Recent numerical studies of the gDNLS for a range of values of $\sigma \in (1, 2]$ indicate that finite time blow-up may occur and give precisely the local structure of the blow-up solutions in terms of the blow-up rate and a universal profile Q depending only on the strength of the nonlinearity σ. This complex valued profile is a solution of a nonlinear elliptic ODE with 2 real valued parameters a and b with an integral constraint.

Using methods of asymptotic analysis, we study the deformation of this profile and the parameters as the nonlinearity σ tends to 1. We find that Q tends to the lump soliton of DNLS while the parameter a tends to 0 like a power law in $(\sigma - 1)$. We compare our results to a numerical integration of the nonlinear elliptic ODE using continuation methods. This is a joint work with G. Simpson and C. Sulem.