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Existence and uniqueness of minimizers of general least gradient problems

Motivated by problems arising in conductivity imaging, we prove existence, uniqueness, and comparison theorems - under certain sharp conditions - for minimizers of the general least gradient problem

$$\inf_{u \in BV_f(\Omega)} \int_\Omega \varphi(x, Du),$$

where $f : \partial\Omega \to \mathbb{R}$ is continuous,

$$BV_f(\Omega) := \{v \in BV(\Omega) : \quad \forall x \in \partial\Omega, \quad \lim_{r \to 0} \textit{ ess sup}_{y \in \Omega, |x-y|<r} |f(x) - v(y)| = 0 \}$$

and $\varphi(x, \xi)$ is a function that, among other properties, is convex and homogeneous of degree 1 with respect to the ξ variable. In particular we prove that if $a \in C^{1,1}(\Omega)$ is bounded away from zero, then minimizers of the weighted least gradient problem $\inf_{u \in BV_f} \int_\Omega a|Du|$ are unique in $BV_f(\Omega)$. We construct counterexamples to show that the regularity assumption $a \in C^{1,1}$ is sharp, in the sense that it can not be replaced by $a \in C^{1,\alpha}(\Omega)$ with any $\alpha < 1$. This is joint work with Amir Moradifam and Adrian Nachman.