Model Theory
Théorie des modèles
(Org: **Omar Sanchez** and/et **Patrick Speissegger** (McMaster))

GAL BINYAMINI, University of Toronto
Bezout-type theorems for differential fields

We consider the following problem: given a set of algebraic conditions on an n-tuple of functions and their first l derivatives, admitting finitely many solutions in a differentiably closed field, give an upper bound for the number of solutions. I will present estimates in terms of the degrees of the algebraic conditions, or more generally the volumes of their Newton polytopes (analogous to the Bezout and BKK theorems). The estimates are singly-exponential with respect to n, l and have the natural aymptotic with respect to the degrees or Newton polytopes. This result sharpens previous doubly-exponential estimates due to Hrushovski and Pillay.

I will give a brief overview of the geometric ideas behind the proof. If time permits I will also discuss some diophantine applications.

CHRIS EAGLE, University of Toronto
Saturation and elementary equivalence of commutative C-algebras*

I will describe recent results about the model theory of commutative unital C*-algebras in the framework of continuous logic. It is well-known that the commutative unital C*-algebras are exactly the algebras of continuous $\mathbb{C}$-valued functions on compact Hausdorff spaces, so model-theoretic properties of the C*-algebra $C(X)$ are related to the topology of the space X. In this talk I will give examples of the topological properties implied by $C(X)$ being countably saturated, and also describe all of the complete theories of the algebras $C(X)$ where X is 0-dimensional. This is joint work with Alessandro Vignati.

JAMES FREITAG, University of California Berkeley
Varieties and isogeny classes of elliptic curves

Fix some transcendental numbers $\bar{a} \in \mathbb{C}^n$. Let $Iso(\bar{a})$ denote the isogeny class of $\bar{a}$, viewing $\mathbb{A}^n$ as the moduli space of products of elliptic curves. Call a variety weakly special if it is defined by a Boolean combination of modular polynomial relations and equations of the form $x_i = b$.

If $V \subseteq \mathbb{C}^n$ is a non-weakly-special variety, then $V \cap Iso(\bar{a})$ is not Zariski dense in V. We will discuss how to use differential algebra to give an effective upper bound on the degree of the Zariski closure of $V \cap Iso(\bar{a})$. In the case that one knows that $V \cap Iso(\bar{a})$ is zero-dimensional (e.g. V is a curve or V contains no weakly special varieties), this gives an effective bound on the number of points in the intersection.

The proofs use intersection theory in jet spaces and various notions from geometric model theory.

BRADD HART, McMaster University
Model theory and operator algebras

With the recent introduction of continuous logic, the connection between model theory and operator algebras which had been hinted at in early work of Robinson and later work of Henson could come to fruition. I will give an overview of the main themes in this growing field which include McDuff's problem and the elementary theory of II_1 factors, the interaction of classification of nuclear C*-algebras and omitting types, and various model theoretic construction principles. I will focus on the current state of affairs and the main open problems.

PHILIPP HIERONYMI, University of Illinois
Expansions of the ordered additive group of real numbers by two discrete subgroups

Let $a \in \mathbb{R}$. We consider the following structure $\mathcal{R}_a := (\mathbb{R}, <, +, \mathbb{Z}, \mathbb{Z}a)$. Although it is well known that $(\mathbb{R}, <, +, \mathbb{Z})$ has a decidable theory and other desirable model theoretic properties (arguably due to Skolem and later rediscovered independently by Weispfenning and Miller), the question whether the theory of $\mathcal{R}_a$ is decidable even for some irrational number a has been open for a long time. The interest in these structures arises among other things from the observation that the structure $\mathcal{R}_a$ codes many of the Diophantine properties of a. In this talk, I will show that when a is quadratic, the theory of $\mathcal{R}_a$ is decidable. The proof of this statement depends crucially on the periodicity of the continued fraction expansion of a and combines classical tools from the theory of Diophantine approximations (in particular, Ostrowski representations) with Büchi's celebrated theorem about the decidability of the monadic second order theory of one successor.

MATTHEW LUTHER, McMaster University
Instability of Asymptotic Cones of Symmetric Spaces

In continuous logic, asymptotic cones of a pointed metric space (X, p, d) are ultraproducts $\prod(X, p, d/n)$ where d/n is a rescaling of the metric d. Asymptotic cones of symmetric spaces are objects with additional structure called R-buildings. I will discuss the definability of this additional structure over the pure pointed metric space, and how this leads to instability.

MARYANTHE MALLIARIS, University of Chicago
Saturation of ultrapowers and Keisler's order

Keisler's order is a large-scale classification program in model theory which gives a means of comparing the complexity of theories via saturation of regular ultrapowers. The order was long thought to have

five or six classes. The talk will present a recent theorem of Malliaris and Shelah revising this picture.

JANA MARIKOVA, Western Illinois University
Measuring definable sets in o-minimal structures

We introduce a measure on the strongly bounded definable sets in an o-minimal expansion of a real closed field which takes values in an ordered semiring, assigns to intervals their length, and is invariant under a change of variables formula. The construction can be modified in the case when the value group of the standard valuation is of rank one to yield a measure on all the bounded definable sets. This is joint work with M. Shiota.

RAHIM MOOSA, University of Waterloo
Differential fields with free operators

Over the last couple of years, Tom Scanlon and I have developed the model theory of fields equipped with what we called "free operators"; a very general framework that includes derivations and endomorphisms as particular cases. The formalism allows for the consideration of several free operators at once, so for example difference-differential fields, but the model companion was only proved to exist for the theory that imposes no functional equations among the operators. In particular, the theory can not ask the operators to commute. In general this is necessary (a pair of commuting endomorphisms has no model companion) but in many cases of interest it is not. In this talk I will discuss recent joint work with Omar Leon Sanchez that considers the theory of fields equipped with several commuting derivations plus free operators that commute with the derivations.

RONNIE NAGLOO, Graduate Center, CUNY
Internality and the Painlevé property for order one equations

In this talk, we look at the relationship between the Painlevé property for first order algebraic differential equations, as defined (and studied) by Muntingh and Van der Put, and the model theoretic notion 'internality' for definable sets in a differentially closed field of characteristic 0.

SANJAY PATEL, McMaster University
o-minimality for quasi-analytic algebras of functions of one variable

We will discuss how, given a class of functions of one variable satisfying certain conditions, one might construct classes of functions of several variables which will be definable in an o-minimal and model-complete expansion of the real field. The conditions include being a "quasi-analytic" algebra, being analytic except possibly at one point, and possessing a suitable complete function space topology.

ATHIPAT THAMRONGTHANYALAK, OSU
Continuous Definable Skolem Functions in O-minimal Structures

It is well-known that o-minimal expansions of real closed fields admit definable Skolem functions; additionally, by cell decomposition theorem, these functions are piecewise continuous. Unfortunately, we cannot find continuous Skolem functions for every definable family. This leads to the question what is a sufficient condition (on the definable family) of the existence of such functions. In this talk, we consider definable families as set-valued maps and prove an analogue of Michael's Selection Theorem in o-minimal expansions of real closed fields. This theorem provides an answer to the above question.