Financial Mathematics
Mathématiques financières
(Org: **Minsuk Kwak** and/et **Traian Pirvu** (McMaster))

ALEX BADESCU, UQAM
Derivative pricing with Non-Gaussian GARCH Models and their Diffusion Limits

We investigate the weak convergence of a non-Gaussian GARCH model together with an application to the pricing and hedging of European style options determined using different stochastic discount factors (SDF), such as discrete-time Girsanov transformations and exponential affine SDFs. Applying these changes of measure to asymmetric GARCH models sampled at increasing frequencies, we obtain risk neutral families of processes which converge to the same bivariate diffusion, which is no longer a standard Hull-White stochastic volatility process. Additionally, it differs from the one obtained by applying the standard minimal martingale measure in continuous time to the diffusion limit of the GARCH family of processes under the physical measure. We show that for skewed innovations, this risk neutral diffusion limit exhibits a non-zero market price of volatility risk which is proportional to the market price of the equity risk, where the constant of proportionality depends on the skewness and kurtosis of the underlying distribution. Our theoretical results are further supported by extensive numerical simulations.

JOE CAMPOLIETI, Wilfrid Laurier University
Some functionals for solvable diffusion processes with applications to financial modelling and option pricing

We discuss the derivation of closed-form formulae for the distribution and expected value of some functionals of new dual families of solvable diffusion processes that arise from Doob space-time transformations. In particular, we obtain analytical formulae for various transition densities, first-hitting time distributions, joint distributions of extrema, as well as the distribution and expected values of functionals of occupation times of the solvable diffusion processes. We present some numerical applications of the models to the risk-neutral pricing of barrier, lookback and occupation-time options. We conclude with a discussion of future extensions and applications.

MATT DAVISON, Western University
Modeling Energy Spreads with a Novel Mean-reverting Stochastic Process

Joint work with Hashem Moosavi, PhD Candidate in Statistics, Western University

The spread between two related energy prices is a very important quantity throughout energy finance. Of particular interests are the spread between two different energy types, location spreads, and calendar spreads.

At times it is appropriate to consider the spread as a distinct process from the underlying price processes which can be modelled directly. We introduce a new mean-reverting random walk, derive its continuous stochastic differential equation and obtain some analytical results about its solution. This new mean-reverting process is compared with the Vasicek process and its advantages discussed. Since the analytical transition density does not exist for this nonlinear stochastic process, to estimate the model parameters, the local linearization method is deployed. We apply this method to empirical data for modeling the spread between West Texas Intermediate (WTI) crude oil and West Texas Sour (WTS) crude oil.

PATRICE GAILLARDETZ, Concordia
Valuation of Equity-Linked Products in the Presence of Policyholder Surrender Using Risk Measures

Throughout the past couple of decades, the surge in the sale of equity-linked products has led to many discussions on the valuation of surrender options embedded in these products. However, most studies treat such options as American/Bermudian style options. In this presentation, a different approach is presented where only a portion of the policyholders react optimally,

due to the belief that not all policyholders are rational. Through this method, a probability of surrender is found and the product is partially hedged by iteratively reducing the measure of risk to a non-positive value. To demonstrate this, the initial value of the partial hedge for an equity-linked product is found under a bivariate equity/interest model with a copula based dependence structure. A numerical example is presented in order to demonstrate some of the dynamics of this valuation method.

This is joint work with Mehran Moghtadai, MSc.

HUAN (GEORGIA) GAO, Western

CODY HYNDMAN, Concordia University

Optimal measure transformation problems for defaultable bonds, futures prices and forward prices

We associate the price of a defaultable bond with an optimal measure transformation problem. The optimal measure transformation problem is closely related to decoupled non- linear forward-backward stochastic differential equation (FBSDE). In the default-free case we prove the equivalence of the optimal measure transformation problem and an optimal stochastic control problem of Gombani and Runggaldier (Math. Financ. 23(4):659-686, 2013) for default free bonds in the framework of quadratic term structure models. The measure which solves the optimal measure transformation problem is the forward measure. These connections explain why the forward measure transformation employed in the FBSDE approach of Hyndman (Math. Financ. Econ. 2(2):107-128, 2009) is effective. We obtain explicit solutions to FBSDEs with random terminal conditions and with jumps in affine term structure models and quadratic term structure models, which extend Hyndman (Math. Financ. Econ. 2(2):107-128, 2009). The futures price and the forward price of a risky asset are also considered in the framework of optimal measure transformation problems. Joint work with Renjie Wang

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SEBASTIAN JAIMUNGAL, University of Toronto

A Mean-Field Gama Approach To Optimal Execution

This paper introduces a mean field game framework for optimal execution with continuous trading. We generalize the classical optimal liquidation problem to a setting where, in addition to the major agent who is liquidating a large portion of shares, there are a number of minor agents (high-frequency traders (HFTs)) who detect and trade along with the liquidator. Cross interaction between the minor and major agents occur through the impact that each trader has on the drift of the fundamental price. As in the classical approach, here, each agent is exposed to temporary price impact and they attempt to balance their impact against price uncertainty. In all, this gives rise to a stochastic dynamic game with mean field couplings in the fundamental price. We obtain a set of decentralized strategies using a mean field stochastic control approach and explicitly solve for an optimal control up to the solution of a deterministic fixed point problem. As well, we present some numerical results which illustrate how the liquidating agents trading strategy is altered in the presence of the HFTs, and how the HFTs trade to profit from the liquidating agents trading.

PETAR JEVTIC, McMaster University

Assessing the solvency of insurance portfolios via a continuous-time cohort model

This paper evaluates the solvency of a portfolio of assets and liabilities of an insurer subject to both longevity and financial risks. Liabilities are evaluated at fair-value and, as a consequence, interest-rate risk can affect both the assets and the liabilities. Longevity risk is described via a continuous-time cohort model. We evaluate the effects of natural hedging strategies on the risk profile of an insurance portfolio in run-off. Numerical simulations, calibrated to UK historical data, show that systematic longevity risk is of particular importance and needs to be hedged. Natural hedging can improve the solvency of the insurer, if interest-rate risk is appropriately managed. We stress that asset allocation choices should not be independent of the composition of the liability portfolio of the insurer.

MINSUK KWAK, Department of Mathematics and Statistics, McMaster University
Cumulative prospect theory with skewed return distribution

We investigate a one-period portfolio optimization problem of a cumulative prospect theory (CPT) investor with multiple risky assets and one risk-free asset. The returns of multiple risky assets follow multivariate generalized hyperbolic (GH) skewed t distribution. We obtain a three-fund separation result of two risky portfolios and risk-free asset. Furthermore, we reduce the high dimensional optimization problem to two 1-dimensional optimization problems and derive the optimal portfolio. We show that the optimal portfolio composition changes as some of investor-specific parameters change. It is observed that the consideration of skewness of stock return distribution has considerable impact on the distribution of CPT investor's wealth deviation, and leads to less total risky investment.

ROMAN MAKAROV, Wilfrid Laurier University
Modelling Default Risk with Occupation Time

We study a structural credit risk model based on the occupation time. Default occurs when the firm value process spends a given amount of time under an exhaust barrier for the first time. We demonstrate that the model includes the Merton model and the Black-Cox model as special limiting cases.

ADAM METZLER, Wilfrid Laurier University
Regulatory Concerns Related to CoCo Bonds - Mathematical Modeling

A contingent convertible (CoCo) bond begins life as subordinated debt, but converts into common equity when the issuing institution begins to experience financial distress. The rigorous treatment of contingent capital in the academic literature remains in its infancy, and several important questions remain unaddressed. For instance it is not clear (i) whether or not CoCo investors will have incentives to short the issuing institution's stock when conversion is imminent (a serious concern for regulators) or (ii) how much an objective conversion trigger (i.e. one allowing for regulatory discretion) would add to the cost of contingent capital relative to a purely objective trigger (e.g. one based on the firm's Tier 1 equity falling below 4.5% of its risk-weighted assets). In this talk we present a structural model that can be used to gain insights into these issues. The model allows for the market price of the firm's stock to temporarily deviated from its fundamental value (incorporating the impact of large scale short-selling) and allows for the conversion time to be the first event time in a Cox process with intensity driven by the firm's asset value (incorporating the uncertainty inherent in a subjective trigger). Numerical valuation of CoCos in the model will be discussed and numerical results, calibrated to balance-sheet data for Canadian institutions, will be presented. The talk is based on the PhD thesis of Jingya Li (Applied Mathematics, University of Western Ontario).

BRUNO REMILLARD, HEC Montreal
Option pricing in a discrete time model for the limit order book

In this talk we build a discrete time model for the structure of the limit order book, so that the price per share depends on the size of the transaction. We deduce the value of a portfolio when the investor trades using market orders and a bank account with different interest rates for lending and borrowing. In this setting, we find conditions to rule out arbitrage and solve the problem of pricing and hedging an European call and put options.

DAVID SAUNDERS, University of Waterloo
Lower Tail Independence of Diffusion Hitting Times

The coefficient of tail dependence is a quantity that measures how extreme events in one component of a bivariate copula depend on extreme events in the other component. It is well known that the Gaussian copula has zero tail dependence, a shortcoming for its application in credit risk modeling and quantitative risk management in general. I will discuss recent

work showing that this property is shared by the joint distributions of hitting times of bivariate (uniformly elliptic) diffusion processes.

KRISTINA SENDOVA, University of Western Ontario
On a perturbed dual risk model with dependence between inter-gain times and gain sizes

Dual risk models may be used to model the revenue process of a company with constant expense rate and occasional gains. In this paper, we consider a dual risk model with both inter-gain distribution and expense rate depending on the size of the previous gain. Also, we assume that the surplus process is perturbed by a Brownian motion. Exact solutions for the Laplace transform and the first moment of the time to ruin with an arbitrary gain-size distribution are obtained. Applications with numerical illustrations are provided to examine the impact of the dependence structure and the perturbation.

ANATOLIY SWISHCHUK, University of Calgary
Volatility Derivatives: History, Ideas and Developments

The talk is devoted to the volatility derivatives, in particular, variance, volatility, covariance and correlation swaps, to name a few. It will include the history and ideas behind the modeling and pricing of volatility derivatives, and literature review. We will also talk about the specific methods and results in this areas, new developments and some challenges and open problems.

XINGHUA ALAN ZHOU, Western University
Misrepresentation and Capital Structure: Quantifying the Impact on Corporate Security Value

Securities class actions typically involve some misrepresentation by a firm that overstates its true value. Upon misrepresentation disclosure, the observed share price drop is used to assess potential damages to shareholders. Using a capital structure model and leveraging a relationship between equity and firm value, we use observable equity information to determine firm value, including the value of other capital structure securities, and hence the effect of misrepresentation on these securities. We find that the misrepresentation impact on debt value depends on firm leverage and debt seniority. Generally, the debt for higher-leverage firms is more sensitive to the misrepresentation impact than for lower-leverage firms and junior debt is more affected by fraud than senior debt.

Our findings have important consequences for damages assessment and allocation of settlement awards in securities class actions. In some jurisdictions damages awarded are net of any hedge or risk-limitation transaction. Since corporate securities such as bonds, stocks and warrants are often held in portfolios for hedging purposes, measuring the effect of misrepresentation on all of the firm's issuances is essential to accurately computed damages awards. Additionally our approach provides a consistent methodology for computing damages for securities that do not trade on public markets. A case study of a recent securities class action illustrates our methodology.