Frames, Fractals, Tiling, and Wavelets, in Connection with the Fuglede's Conjecture
Repères, fractales, pavages et ondelettes, par rapport à la conjecture de Fuglede

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TRUBEE DAVISON, University of Colorado

Generalizing the Kantorovich Metric to Projection Valued Measures: With an Application to IFS

Given a compact metric space X, the collection of Borel probability measures on X can be made into a compact metric space via the Kantorovich metric (Hutchinson in Indiana Univ. Math. J. 30(5):713–747, 1981). We partially generalize this well known result to projection valued measures. In particular, given a Hilbert space $\mathcal{H}$, we consider the collection of projection valued measures from X into the projections on $\mathcal{H}$. We show that this collection can be made into a complete and bounded metric space via a generalized Kantorovich metric. However, we add that this metric space is not compact, thereby identifying an important distinction from the classical setting. We have seen recently that this generalized metric has been previously defined by F. Werner in the setting of mathematical physics (Werner in J. Quantum Inf. Comput. 4(6):546–562, 2004). To our knowledge, we develop new properties and applications of this metric. Indeed, we use the Contraction Mapping Theorem on this complete metric space of projection valued measures to provide an alternative method for proving a fixed point result due to P. Jorgensen (see Adv. Appl. Math. 34(3):561–590, 2005; Operator Theory, Operator Algebras, and Applications, pp. 13–26, Am. Math. Soc., Providence, 2006). This fixed point, which is a projection valued measure, arises from an iterated function system on X, and is related to Cuntz Algebras.

DORIN DUTKAY, University of Central Florida

Tiling and scaling properties of spectra of fractals

We present some recent results on the tiling properties of the Fourier frequencies associated to a Fourier basis for a fractal measure and some number theoretic questions related to the spectra of the Jorgensen-Pedersen Cantor set.

KYLE HAMBROOK, University of British Columbia

Explicit Salem Sets

Fractals with matching Hausdorff and Fourier dimension are called Salem sets. Salem sets represent an important general setting for the Fourier restriction problem. Salem sets are ubiquitous amongst random sets, but explicit examples of Salem sets are exceedingly rare. I will discuss my recent result which generalizes theorems of Kaufman and Bluhm and yields new explicit families of Salem sets. I will also discuss applications to metrical Diophantine approximation and directions for further study.

DEGUANG HAN, University of Central Florida

Frames, dilations and related problems

Motivated by the Banach space nature of framing dilation established by D. Han, P. Casazza and D. Larson, we establish a general dilation theory for operator valued measures and maps between von Neumann algebras that include maps that are not necessarily completely bounded. Our results lead to some new connections between frame theory and operator algebras. This is a joint work with D. Larson, Bei Liu and Rui Liu.

KATHRYN HARE, Dept. of Pure Mathematics, University of Waterloo

Hausdorff and packing measures of balanced Cantor sets

For central Cantor sets, such as the classical middle-third Cantor set, it is well known how to calculate their Hausdorff and packing dimensions. This is also known for Cantor-like sets associated with decreasing sequences. In this talk we will discuss this problem for a more general class of perfect, totally disconnected sets that we call balanced Cantor sets. We also give bounds on the h-Hausdorff and h-packing measures for these sets and we see how these compare with the measures of all other compact sets whose complement consists of a collection of disjoint open intervals of the same lengths as the original set.

MIHALIS KOLOUNTZAKIS, University of Crete

Riesz bases of exponentials for domains that tile multiply with a lattice

Suppose Ω is a bounded, measurable set in Euclidean space which tiles space at some positive integer level k (almost every point is covered exactly k times) when translated at the locations of a lattice Λ. Grepstad and Lev showed that such domains have a Riesz basis of exponentials, provided they have a boundary of 0 measure. Their method used results of Matei and Meyer on quasicrystals. We prove the same result (without the requirement for null boundary) using a different argument based mostly on linear algebra. Open problems are discussed.

CHUN-KIT LAI, San Francisco State University

Fuglede's and generalized Fuglede's conjecture on $\mathbb{R}^1$

Fuglede's conjecture stated that translational tiles and spectral sets are equivalent. This conjecture was disproved by Tao, later by Kolountzakis and Matolcsi on $\mathbb{R}^d$, $d \geq 3$. The conjecture now remained open when $d = 1, 2$. In this talk, we reviewed some recent work on Fuglede's conjecture on $\mathbb{R}^1$ and proposed a generalized one which includes all fractal spectral measures.

FRANKLIN MENDIVIL, Acadia University

Hausdorff and packing measures of Cantor sets associated with series

Given a convergent infinite series of positive terms a_n, the set of all possible subsums forms a perfect and compact set. If in addition the series satisfies a fast decay condition, the resulting set, C_a, is totally disconnected.

Manual Morán studied these C_a, extended the construction to $\mathbb{R}^d$, and gave some results on the Hausdorff dimension and measure. In this talk, I will discuss our extension of his construction and present results regarding the Hausdorff and packing dimension and measure of C_a. I will also present some results concerning the existence of subsets of C_a of a given (smaller) dimension and measure. In particular, we show that for all $d < dim(C_a)$ there exist a subsequence b_n of a_n with $dim(C_b) = d$. This is joint work with Kathryn Hare and Leandro Zuberman

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JASON PHILLIPS, Wright State University

BARIS UGURCAN, Western University, Canada.

Whitney-type Problems and Haar Expansions on the Sierpinski Gasket

Whitney extension problem is studied in classical analysis where one wants to extend the data on a finite set. In this setting, the data (often function values and certain derivative values) is extended in such a way that it will minimize some prescribed norm. In this work, we initiate the study of such problems on fractals. We are interested in minimizing two Sobolev types of norms and expressing the energy of the minimizer by using Haar functions in certain cases. The first one is $\mathcal{E}_\lambda(f) = \mathcal{E}(f, f) + \lambda \int f^2 d\mu$, where $\mathcal{E}$ denotes the self-similar energy on the Sierpinski Gasket. For $\lambda = 0$, this corresponds to minimizing the energy whereas for $\lambda > 0$ we are minimizing the analog of H^1-Sobolev norm in the Euclidean space. We prove the existence and uniqueness of a minimizer and construct the minimizer explicitly in terms of the resolvent of the Laplacian. One of the specific sets that we consider the Whitney-type problem is the bottom row of a graph approximation of the Sierpinski Gasket. In this case, we express the energy of the minimizer in a specific form using Haar functions. The second type of Sobolev norm we consider is $\int_{SG} |\Delta f(x)|^2 d\mu(x)$ which is the analog of the Sobolev H^2-norm. We are interested in the cases of self-similar measure and

Kusuoka measure. Some of our results extend to the Kigami's post-critically finite (PCF) fractals. This is joint work with Pak-Hin Li, Nicholas Ryder and Robert S. Strichartz.

JÓZSEF VASS, University of Waterloo
On the Exact Convex Hull of IFS Fractals

Fuglede's Conjecture relates spectral sets to tiles of positive Lebesgue measure, such as connected self-affine tiles, further underlining their relevance. The characterization of connected self-affine tiles has evoked plenty of recent interest, as well as that of IFS fractals in general. Connectedness is hinted to be related to the convex hull problem through self-contacting binary fractal trees. This talk surveys the current state of the convex hull problem, and introduces novel results and approaches. The objective is to determine the exact convex hull, not mere approximations, in order to potentially aid the resolution of other theoretical problems.

ERIC WEBER, Iowa State University
Fourier Frames for the Cantor-4 Set

Jorgensen and Pedersen constructed a Cantor type set consisting of numbers whose base four expansion contains digits 0 and 2. They showed that this set possesses a Fourier basis. We demonstrate that weighted Fourier Frames can be obtained for this set by making the set bigger, constructing a basis for the bigger set, and then projecting the basis onto the Cantor set. This is joint work with Gabriel Picioroaga.