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Generalizing the Kantorovich Metric to Projection Valued Measures: With an Application to IFS

Given a compact metric space X, the collection of Borel probability measures on X can be made into a compact metric space via the Kantorovich metric (Hutchinson in Indiana Univ. Math. J. 30(5):713–747, 1981). We partially generalize this well known result to projection valued measures. In particular, given a Hilbert space $\mathcal{H}$, we consider the collection of projection valued measures from X into the projections on $\mathcal{H}$. We show that this collection can be made into a complete and bounded metric space via a generalized Kantorovich metric. However, we add that this metric space is not compact, thereby identifying an important distinction from the classical setting. We have seen recently that this generalized metric has been previously defined by F. Werner in the setting of mathematical physics (Werner in J. Quantum Inf. Comput. 4(6):546–562, 2004). To our knowledge, we develop new properties and applications of this metric. Indeed, we use the Contraction Mapping Theorem on this complete metric space of projection valued measures to provide an alternative method for proving a fixed point result due to P. Jorgensen (see Adv. Appl. Math. 34(3):561–590, 2005; Operator Theory, Operator Algebras, and Applications, pp. 13–26, Am. Math. Soc., Providence, 2006). This fixed point, which is a projection valued measure, arises from an iterated function system on X, and is related to Cuntz Algebras.