BARIS UGURCAN, Western University, Canada.
Whitney-type Problems and Haar Expansions on the Sierpinski Gasket

Whitney extension problem is studied in classical analysis where one wants to extend the data on a finite set. In this setting, the data (often function values and certain derivative values) is extended in such a way that it will minimize some prescribed norm. In this work, we initiate the study of such problems on fractals. We are interested in minimizing two Sobolev types of norms and expressing the energy of the minimizer by using Haar functions in certain cases. The first one is $\mathcal{E}_\lambda(f) = \mathcal{E}(f,f) + \lambda \int f^2 d\mu$, where $\mathcal{E}$ denotes the self-similar energy on the Sierpinski Gasket. For $\lambda = 0$, this corresponds to minimizing the energy whereas for $\lambda > 0$ we are minimizing the analog of H^1-Sobolev norm in the Euclidean space. We prove the existence and uniqueness of a minimizer and construct the minimizer explicitly in terms of the resolvent of the Laplacian. One of the specific sets that we consider the Whitney-type problem is the bottom row of a graph approximation of the Sierpinski Gasket. In this case, we express the energy of the minimizer in a specific form using Haar functions. The second type of Sobolev norm we consider is $\int_{SG} |\Delta f(x)|^2 d\mu(x)$ which is the analog of the Sobolev H^2-norm. We are interested in the cases of self-similar measure and Kusuoka measure. Some of our results extend to the Kigami's post-critically finite (PCF) fractals. This is joint work with Pak-Hin Li, Nicholas Ryder and Robert S. Strichartz.