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Probing Fundamental Bounds in Hydrodynamics Using Variational Optimization Methods

In the presentation we will review recent results and discuss some emerging research directions concerning application of the modern methods of PDE-constrained optimization to study a class of fundamental problems in mathematical fluid mechanics. These problems concern the sharpness of certain estimates, such as the bounds on the maximum enstrophy growth in 3D flows governed by the Navier-Stokes system, which are intimately related to the question of spontaneous singularity formation, known as the "blow-up" problem. We demonstrate how new insights concerning such problems can be obtained by formulating them as variational PDE optimization problems which can be solved computationally using discretized gradient flows. Vortex states determined in this way represent the most extreme flow behavior allowed for by the Navier-Stokes dynamics and are therefore natural candidates for blow-up. In offering a systematic approach to finding such flow solutions which saturate known estimates, the proposed paradigm provides a bridge between theory and computation. In the presentation we will show a number of new results concerning extreme vortex states in 2D and 3D Navier-Stokes flows, and will discuss their relation to the available theoretical bounds obtained with methods of mathematical analysis. [Joint work with Dr. Diego Ayala]