Modeling and Analysis of the Dynamical Impact of Dissolved Oxygen on Phytoplankton and Zooplankton in a Lake

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YORK UNIVERSITÉ UNIVERSITY redefine THE POSSIBLE.

LAMPS LAboratory of Mathematical Parallel Systems

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Introduction

Harmfulness of algal bloom

Possible reasons for algal bloom

♦ Nutrient
1) Phosphorus
2) Nitrogen
...

♦ Environmental factors:
a) temperature
b) solar radiation
c) light
d) dissolved oxygen (DO)
...

The roles of DO in water body

♦ Influencing the survival of aquatic organisms;

♦ Reflecting the extent of the contaminated water;

♦ Evaluating the quality of drinking water;

...

Our purpose

♦ Study the effect of DO concentration on the growth of phytoplankton and zooplankton;

♦ Obtain some useful information to better understand the mechanism of outbreaks of the bloom.

Mathematical Modeling

How to model

A model is proposed from a conceptual framework in view of the principles of mass balance.

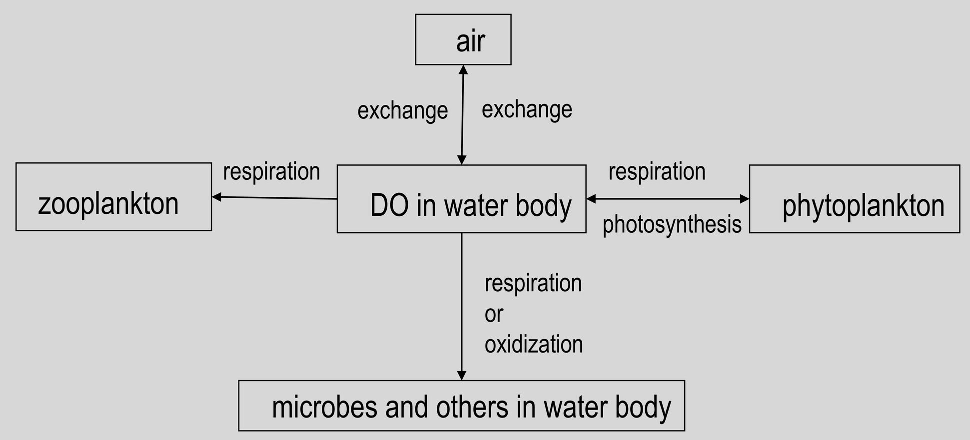

Figure1: Schematic diagram of the dissolved oxygen (DO) model.

Assumptions

➤Zooplankton will be totally dead if DO concentration is below the critical value;

➤The amount of DO for exploitation of microbial respiration et al. is assumed to be proportional to the concentration of DO in water body;

➤The net production of DO from photosynthesis and respiration of phytoplankton is supposed to be proportional to the density of phytoplankton;

➤The amount of DO consumed by zooplankton respiration depends linearly on the density of zooplankton.

Our model

The proposed model with three interacting components: phytoplankton (P), zooplankton (Z) and dissolved oxygen (DO)

$$\frac{dP}{dt} = rP\left(1-\frac{P}{K}\right) - \frac{mP}{P+P_0}F_1(D)Z$$

$$(1) \qquad \frac{dZ}{dt} = \beta\frac{mP}{P+P_0}F_1(D)Z - dZ$$

$$\frac{dD}{dt} = q(D_2 - D) + \mu_1 P - \mu_2 Z - \mu_3 D$$

where

$$F_1(D) = \begin{cases} 0 & if\ \ D < D_1, \\ \dfrac{\delta(D-D_1)}{D_0+D} & if\ D \geq D_1, \end{cases}$$

and the ecological meanings of all parameters are given in table 1.

Parameters	Meanings
P	The density of phytoplankton
D	The concentration of DO
Z	The density of zooplankton
r	The growth rate of phytoplankton
K	The environmental carrying capacity of phytoplankton
m	The maximal per capita consumption rate
δ	The maximal relative influence ratio of DO on the predation ability of zooplankton
β	The conversion rate of consumed phytoplankton into new zooplankton
P_0	The Michealis-Menton half saturation constant
d	The mortality rate of zooplankton due to natural death as well as higher predation
q	The exchange velocity of DO through air-water interaction
D_0	The half saturation concentration of DO in water
D_1	The critical concentration of DO leading to zooplankton death
D_2	The saturation concentration of DO in water
μ_1	The net growth rate of DO produced by photosynthesis and respiration of phytoplankton
μ_2	The consumption rate of DO by zooplankton through respiration
μ_3	The consumption rate of DO by others which need to consume oxygen in water

Table 1: The meanings of parameters of model.

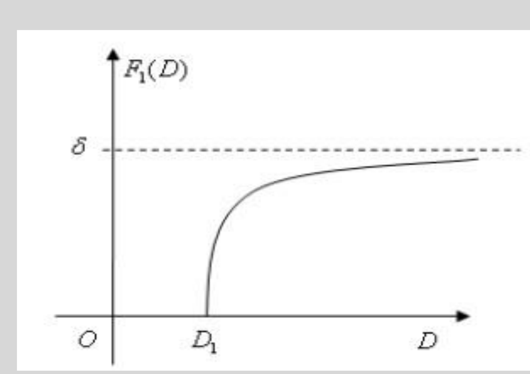

Figure 2: The effect of dissolved oxygen function.

Theoretical Analysis

Existence of equilibrium points

Theorem 1. For system (1), the equilibria $E_1(0,0,\frac{qD_2}{q+\mu_3})$ and $E_2(K,0,\frac{qD_2+\mu_1 K}{q+\mu_3})$ always exist.

If $R_0 \leq 1$, there is no any positive equilibrium. If $R_0 > 1$, there will be at least one positive equilibrium and three positive equilibria at most, where $R_0 = \frac{\beta m\delta K[qD_2+\mu_1 K - D_1(q+\mu_3)]}{d(P_0+K)[qD_2+\mu_1 K+D_0(q+\mu_3)]}$.

Stability of equilibrium points

Theorem 2. For system (1), we have

Equilibrium	R_0<1	R_0=1	R_0>1
E_1	Saddle	Saddle	Saddle
E_2	Stable node	Saddle-node	Saddle
E_3	Not exist	Not exist	Non-saddle

Theorem 3. Let $E^*(P^*, Z^*, D^*)$ be any equilibrium point of the model (1),

(a) if there exists δ satisfying $\frac{d}{\beta m}(1+P_0/K) \leq \delta \leq \frac{d}{\beta m}(1+2P_0/K)$ and $H(\delta) > 0$, then the positive equilibrium E^* of the system (1) is asymptotically stable;

(b) if there exists δ^* satisfying $H(\delta^*) = 0$ and $\left(k_0' - k_2(k_1)'\right)|_{\delta=\delta^*} \neq 0$, then the system (1) will experience a Hopf bifurcation at E^*, where

$$k_0 = \left\{(q+\mu_3)d + \left(\mu_1 d - \frac{\mu_2\beta r(K-2P^*)}{K}\right)\frac{((\beta m\delta - d)P^* - dP_0)^2}{\beta m\delta dP_0(D_0+D_1)}\right\}\frac{rP_0(K-P^*)}{K(P_0+P^*)};$$

$$k_1 = \left\{\frac{(\mu_2\beta+\mu_1)((\beta m\delta - d)P^* - dP_0)^2}{\beta m\delta dP_0(D_0+D_1)} + (q+\mu_3+d)\right\}\frac{rP_0(K-P^*)}{K(P_0+P^*)} - \frac{(q+\mu_3)r(K-2P^*)}{K};$$

$$k_2 = (q+\mu_3) + \frac{rP_0(K-P^*)}{K(P_0+P^*)} - \frac{r(K-2P^*)}{K};$$

$$H(\delta) = k_2 k_1 - k_0.$$

Numerical Simulations

Comparing with a general phytoplankton-zooplankton model

$$\frac{dP}{dt} = rP\left(1-\frac{P}{K}\right) - \frac{mP}{P+P_0}Z$$

$$(2) \qquad \frac{dZ}{dt} = \beta\frac{mP}{P+P_0}Z - dZ$$

Parameters	Reference	Default values
r	0.07-0.28[6]	0.15(day^{-1})
K	108[19]	108(mg/l)
m	0.6-1.4[6]	0.8(dimensionless)
β	0.2-0.5[6]	0.3(dimensionless)
P_0	5.7[19]	15(mg/l)
d	0.001-0.125[9]	0.1(day^{-1})
q	2[9]	2(day^{-1})
D_0	1.4[9]	1.4(mg/l)
D_1	-	1(ml/l)
D_2	6.412-14.621[9]	9(ml/l)
μ_1	1.4-1.8[9]	1.4(day^{-1})
μ_2	0.01-0.36[9]	0.36(day^{-1})
μ_3	-	6(day^{-1})

Table 2: Parameters for the simulations of the models.

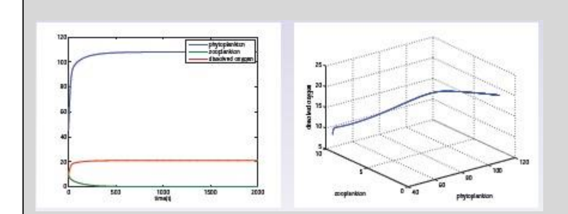

Figure 3: The solution curve and the phase space diagram of system with initial value (45; 10; 8) for δ=0.5.

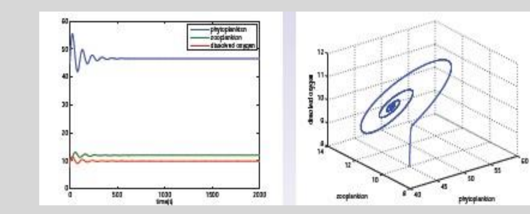

Figure 4: The solution curve and the phase space diagram of system with initial value (45; 10; 8) for δ = 0.7.

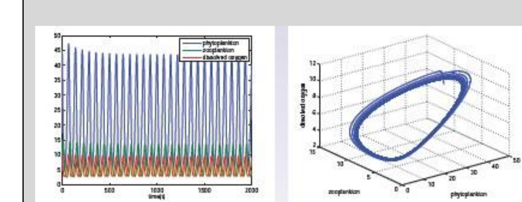

Figure 5: The solution curve and the phase space diagram of system with initial value(45; 10; 8) for δ= 1.2.

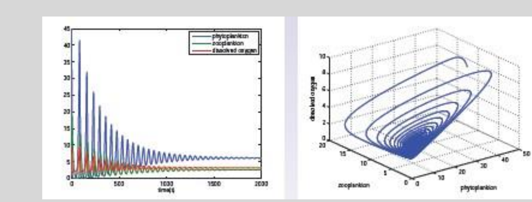

Figure 6: The solution curve and the phase space diagram of system with initial value(45; 10; 8) for δ= 3.

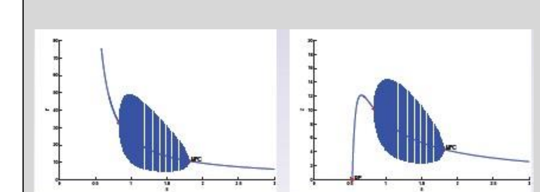

Figure 7: Bifurcation diagram of system , where P and Z are plotted as a function of respectively.

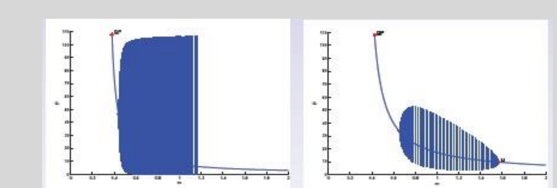

Figure 8: Bifurcation diagrams of system (1) (right) and system (2) (left) with β= 1.2 by selecting m as a bifurcation parameter.

Conclusions

♣Zooplankton must either coexist with phytoplankton or die out in the lake ecosystem;

♣ Zooplankton will become extinct and phytoplankton will have the maximal population if the growth rate of zooplankton is less than the death rate.

♣ Taking the maximum relative influence ratio of dissolved oxygen on the predation ability of zooplankton as a bifurcation parameter, periodic oscillatory behavior is shown within a continuous range of this parameter, which suggests that the algal bloom is likely to occur on different scales.

♣ By comparing with a general phytoplankton-zooplankton model, dissolved oxygen in water body does have an important impact on the dynamical behaviors of phytoplankton and zooplankton and plays a significant role in determining the occurrence and termination of algal blooms.